TECHNIQUE FOR THE STUDY OF MOTION OF THE JOINTS OF THE FOOT

by

MAN-CHUEN YUEN

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Signature of Author Man-Chuen Yuen
Department of Civil and Sanitary Engineering
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Certified by
Thesis Supervisor Robert J. Hansen

Signature of Chairman
of Department Committee
on Graduate Students Myle J. Holley, Jr.

ABSTRACT

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The present study is concerned with the development of a technique for the quantitative study of the range and type of motion in the foot. A scanning system is used to measure the X, Y, Z coordinates of the three target points of the target attached to the bone at each position and the equations of rigid body kinematics are applied to calculate both the translation and rotation. At present, 18 amputated limbs have been tested, and no conclusive results can be drawn yet. The method by itself is rigorous, but the results have to be interpreted with other biological laws in mind.

Thesis Supervisor: Robert J. Hansen

Title: Professor of Structural Engineering

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Summary

A) Objective.

It has been known in the medical field that the motion of the bones in the foot is a complex spactial phenomenon which generally involves both translation and rotation. The object of this research, therefore, is the development of a technique for the quantitative study of the range and type of motion in the foot.

B) Previous work.

Most work that has been accomplished in the past can be classified as only qualitative in nature. The first attempt for the quantitative study of the motion of the foot was the three-camera technique developed by the University of California group. The result of this investigation indicated the enormous amount of data involved and the difficulty in the reduction of data. This same approach was later simplified at M.I.T. by using only two cameras.

C) Theory.

The theory is concerned with the application of equations of kinematics of rigid body to the study of the kinematical motion of the foot. The motion is separated into translation and rotation and they are calculated independently of each other. The rotation is described by the three Euler Angles.

D) Experimental procedure.

For the calculation of translation and rotation of any point

in the bone, the coordinates of these non-colinear points in the bone at each position have to be known. A target with three target points is attached rigidly to the bone to being studied, and the coordinates of the target points at each position are measured by the scanning system. At present manual recording is being done; however, it is anticipated that an automatic readout system can be used which would automatically translate and record the coordinates into punch cards for a computer. The enormous amount of calculation involved necessitates the use of a digited computer for the calculation. A computer program is written for the calculation of translation and rotation of axes and the rotation and translation of any point in the bone with respect to a fixed reference axes.

E) Error Analysis.

The methods in the theory of error are used to estimate the accuracy of the system. The error in translation is approximately ± 5% and the error in rotation should be of the same order of magnitude. The study indicates that in the future more attention should be directed to the establishment of the reference axes and the choice of points to be studied.

F) Conclusion.

So far, only 18 amputated limbs have been tested, and no meaningful conclusive result can be drawn from this data. The method by itself is rigorous, but the result obtained should be interpreted with other biological laws in mind.

Chapter 2.

Introduction

A) Nature of the problem.

Human locomotion with its boundless variability is the product of a multitude of only partially known factors. These factors cannot be interpreted readily within the framework of precise mathematical and physical laws. Nevertheless, the operation of known physical laws can be recognized clearly and evaluated when locomotion events are observed by appropriate methods. Therefore, any errors or inconsistencies, which may be encountered, are the result of inadequancies of the method of observation and misinterpretation of observation rather than that of the law itself. The problem of observation and interpretation is further complicated here as in all biological phenomena by the existence of other laws, not mechanical in nature, which have their share in human locomotion.

The fundamental problem in the study of human locomotion, therefore, is one of observation. Accurate, quantitative methods of observation are not available. It is the purpose of this investigation to develop such a method.

B) Functional anatomy of the human foot.

The profound changes which have taken place in the foot during its adaptation from the quadrupedal to the orthograde type of motion are based upon the requirement of adjusting the center and line of gravity to a small supporting surface. Once gravitational

stresses were so adapted to the area of the support the further problem was to keep them so adjusted that the body could not only stand erect upon it but could also be balanced by muscle action against fluctuations of the line of gravity. Furthermore, the alternating bipedal gait made it necessary to develop in the foot a means of propulsion as well as a means of restraint to maintain the body in equilibrium or during locomotion.

Although there are a great number of articulations in the foot, from a functional point of view, there are but four joints or groups of joints which are of significance.

First, the ankle, the articulation between the talus and the lower ends of the tibia and fibula, makes motion possible in the sagittal plane which is plantar and dorsal flexion of the foot. It is largely by movement at this joint that the organism adjusts the line of gravity during standing and provides for propulsion and restraint in the sagittal plane during gait.

Second, the subtalar joint between the talus and calcaneus is chiefly responsible for motion of the foot in the frontal plane which is inversion and eversion. By movements of this joint the organism adjusts the line of gravity from side to side and adjusts the plane of the sole of the foot to variations and irregularities of the ground surface during propulsion. In addition, there is also slight movement possible in the sagittal plane at this joint.

Third, the midtarsal joint consists of two articulations. Medially is the joint between the head of the talus and the scaphoid

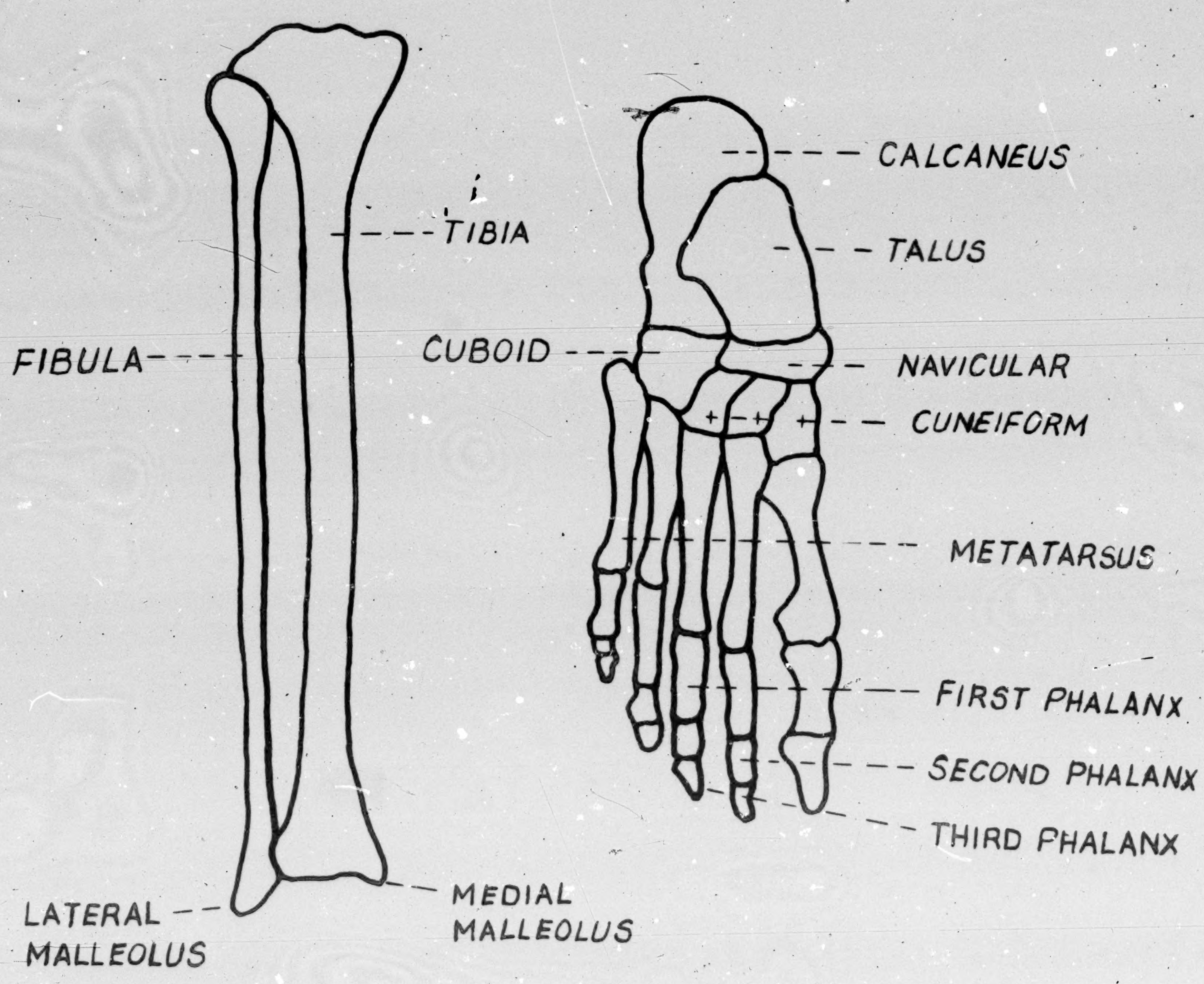

Fig. 1. Bones of Right Leg.

and lateral to this in roughly the same frontal plane is the joint between the anterior process of the calcaneus and the cuboid. This joint really allows motion in three planes and enables the front part to maintain close contact with the ground as the back part carries out its pronatory and supinatory or abductory and adductory movements.

Fourth are the metatarsophalangeal joints between the heads of the five metatarsals and the corresponding proximal phalanges. These joints permit motion between the toes and the rest of the foot in the sagittal plane. Thismovement permits the fine adjustments between the toes and forefoot which take place during gait as the body weight is thrown forward onto the ball of the foot at take-off or during standing with the heel raised off the ground.

The remaining joints between the tarsals and metatarsals, as well as those between the phalanges, are thought to have little significance so far as functioning of the foot is concerned. All the functional requirements necessary to maintain the line of gravity within the bands of the supporting surface formed by the foot appear to be served by the ankle, the subtalar, the midtarsal and the metatarsophalangeal joints. Deductions have been made concerning the relation of these geometrical differences to the observed variations in the function of feet, but virtually no attempt has been made to obtain quantitative data on the mechanical functions and to correlate these with the geometrical characteristics of the articular structures. Studies of a qualitative nature have been published

which have attempted to locate the axes of rotation and some of the displacement characteristics of the bones comprising the joints of the foot and ankle. It is apparent, however, that in order to understand the complex mechanism of the human foot and the many functional disarrangements which cause pain and disability, quantitative data on the mechanical functions of a large number of feet must be available.

C) Historical background.

An extensive literature concerning both the normal foot and its abnormalities has accumulated in the medical, anatomical and anthropological literature. No attempt is made here to review this literature completely. In general, it may be said that studies of the foot have included:

1) Descriptions of anatomical variations in the skeletal and tendinous structures which have been observed from time to time.
2) Studies of the comparative anatomy of the feet of primates and men.
3) Studies of the differences of the feet of primitive people who have never worn shoes with the feet of the people who have always worn shoes.
4) Clinical studies of abnormal feet and their treatment.
5) Surveys of men in the armed forces including recruits at the time of induction and during basic training as well as of veterans who have developed symptoms while on active duty.

6) Studies of the distribution of forces on the plantar surface of the foot and the influence of shoes and supporting apparatus on the distribution of these forces.

7) Experiments on postmortem material designed to investigate the relative importance of ligamentous structures and of muscle action in the maintenance of the normal arches of the foot.

8) Roentgenographic studies including cinefluorography of feet with and without weight-bearing and during gait.

All these studies have been rather inconclusive since they have failed to attack the real crux of the problem, that is, the mechanical function of the foot. The complex nature of the motions of the foot and ankle together with the enormous variation, which is characteristic of human feet as it is of all biological structures, makes any quantitative study of foot motion a formidable task. Attempts have been made by a few workers to determine the axes and planes of motion of the bones of the foot by both mechanical and photographic methods. These investigations may be summarized as follows:

Manter(1) in a study of the movements of the subtalar and transverse tarsal joints in postmortem limbs determined the planes of motion of these joints in 16 links using a mechanical process. The process involved putting the specimen between two flat glass plates and attaching four pointers to the bone with two in contact at all time with each glass plate. The pointers would trace circular curves on the plates

(1) J. T. Manter, "Movements of the Subtalar and Transverse Tarsal Joints", The Anatomical Record, Vol. 80, No. 4, August 1941.

as the bone was being rotated. The line joining the centers of the circles was assumed to be the axis. His results seem to be quite sensitive and their precision may be questioned.

Barnett and Napier[2] studied the axis of rotation of the ankle joint in postmortem material correlating this with the shape of the talus and the mobility of the fibula. No detail or even brief description of method used is included and it is evident that whatever measurements were made, they were very gross.

Hicks[3] attempted to determine the axes of rotation of all the joints of the foot and their ranges of rotation. The method was to insert a malleable wire into the bone being investigated and then to attach a rod to the free end of the wire. The position of the rod was then adjusted repeatedly until no movement of the rod was visible during motion of the appropriate joint. The direction of the rod was then assumed to represent the axis of rotation. Sixteen limbs were investigated by this method and the axis of movement recorded. An obvious defect in this technique is its failure to take into account the translational movements of these joints.

Direct measurements of the changes in the intermalleolar distance at the ankle during walking and during rotation of the foot at various positions of dorsal and plantar flexion have been made by Close and Inman[4]. A modified caliper was used to measure the changes in the

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(2) C. H. Barnett and J. R. Napier, "The Axis of Rotation at the Ankle Joint in Man. Its Influence Upon the Form of the Talus and the Mobility of the Fibula". Journal of Anatomy, Vol. 86, Cambridge University Press, 1952.

distance between the superficial surfaces of the medial and lateral malleoli. Care was taken to place the caliper at the mid-point of the superficial surface of the lateral malleolus since the rotation of the fibula about its long axis which normally accompanies ankle motion would introduce some error into the measurement of the inter-malleolar distance. It would appear that variations in the selection of this point would introduce considerable errors in this method.

Two techniques for recording data by photographic methods have been used.

The Prosthetic Research Group of the University of California[5] during the investigation of joint movements of the lower extremities in connection with the design of artificial limbs employed living subjects with pins inserted under local anesthesia into the bones of the various segments of the lower extremity. The movement of the free end of these markers was recorded by a three-camera method with each motion picture camera being oriented along one of the three mutually perpendicular planes. This technique has certain inherent disadvantages. First, reducing the data for each position from three movie films is enormously time consuming. Second, as the object moves past the optical axis of each camera, a separate correction for parallax must be made for each point. Finally, a more serious objection to this method lies in the

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(3) J. H. Hicks, "The Mechanics of the Foot. 1. The Joints", Journal of Anatomy, Vol. 87, part 4, Cambridge University Press, 1953.

(4) J. R. Close and V. T. Inman, "The Action of the Ankle Joint", Prosthetic Devices Research Project, Institute of Engineering Research, University of California, Berkeley, April, 1952.

(5) J. R. Close and Inman, V. T., "The Action of the Subtalar Joint", Prosthetic Devices Research Project, Institute of Engineering Research, University of California, Berkeley, May 1953.

fact that the movements recorded are merely those of the marker in space. For data of this type to be significant, there must be a fixed relationship of the marker to the bone to permit reproducible and comparable data to be obtained, and furthermore the marker must be referred to coordinates which have a fixed relationship to the skeleton.

In 1956, Rothstein of MIT[6] investigated the kinematical motions of the joint of the foot by the two-camera stero-photogrammetric method. This method is used widely in the field of aerial photogrammetry for mapping purposes. The method involved setting two cameras along a base line of known length with the optical axes being parallel to each other and perpendicular to the base. The coordinates of the point were then determined from the films of the two cameras. No conclusive results could be drawn from his experiment. However, it was a vast improvement from the three-camera technique, and with the possibility for the automatic reduction of data by means of stereo-plotter, this might be the practical method to be used in a kinematic analysis of the lower extremity.

D) Plan of present study.

The function of the human foot is dependent on a variety of factors including muscle action, the geometry of the bones and their articular surfaces, and the form and placement of the ligaments. All these factors must be taken into consideration in the study of foot

[6] B.S. Thesis, "Kinematic Analysis of the Joints of the Lower Extremity", by H. H. Rothstein, Dept. of Civil and Sanitary Engineering, MIT, June 1956.

function and therefore methods have to be devised to obtain quantitative data concerning each of these factors.

A reasonable starting point would appear to be the study of the motions of the various joints - a kinematic analysis of the foot. By such an analysis, it should be possible to determine the planes and axes of motion of the individual joint with respect to a reference axes or with respect to each other. If these kinematical data are determined on amputated lower limbs, a concurrent quantitative study of the geometry of the bones and of their articular surfaces should be possible by appropriate anthropometric measurements of the disarticulated bones. With these data, a correlative analysis of the motion characteristics of each joint and the geometrical features of the bones comprising these joints will be possible.

As indicated in the previous section, former efforts along these lines have failed to secure quantitative, reproducible data suitable for comparative studies. Methods for recording and reducing the data have been both so inaccurate and time-consuming that virtually no significant progress has been made recently in the study of foot function.

The immediate objective of the present study is the rather formidable task of developing an adequate technique for the study of the kinematics of the foot with special emphasis on rapid methods for the reduction and presentation of the data.

Chapter 3.
Theory

In all previous work, motions were measured by very crude methods, the results were inevitably gross simplifications of complex motions. It was observed in recent years that the motion of the joints or that of one bone with respect to the other was in most cases not a simple planary or purely rotational motion, but a complex space motion which involves both translation and rotation. The present theory is concerned with the application of certain known equations of kinematics of rigid body motion to the study of the kinematical motion of the foot. The only assumption we have to make is that all bones are considered to be rigid bodies. The other thing we have to consider in this study is the presentation of data. As a result, a set of arbitrary orthogonal axes is chosen as reference axes so that all results can subsequently be compared.

A) Kinematics of rigid body motion.

A rigid body is defined as a system of mass points subject to the holonomic constraints that the distances between all pairs of points remain constant throughout the motion. To fix a point in the rigid body, we need only to specify its distances to any three other non-collinear points. Once the positions of those three particles are determined, the constraints fix the positions of all remaining particles.

The degree of freedom for three points are at most $3 \times 3 = 9$ but there exist in addition three constraint equation:

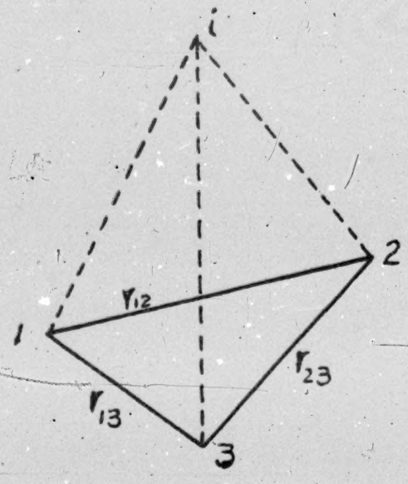


Fig. 2.

$$r_{12} = C_{12}$$

$$r_{23} = C_{23}$$

$$r_{13} = C_{13}$$

where C denotes a constant.

Hence a rigid body in space thus needs only six independent generalized coordinates to specify its configuration, no matter how many particles it may contain.

The problem is further simplified by Chasle's Theorem which states that any general displacement of a rigid body can be represented by a translation plus a rotation. The six coordinates needed to describe the motion have already been formed into two sets in accordance with such a separation. The three cartesian coordinates of a point fixed in the rigid body to describe the translational motion and the three other coordinates (say Eulerian Angles) for the rotational motion about the point.

The entire mechanical problem does indeed split into two, one involving only the translational coordinates, the other only the angular coordinates. The two groups of coordinates will then be completely separated, and the translational and rotational problems can be solved independently of each other.

The present method of investigation is to measure the coordinates of three non-collinear points on a rigid body at each position and then calculate the translation and rotation of any other point of the rigid body.

B) Translational motion.

The translation of the point in question, generally, cannot be measured directly, and a method is resolved in which once the relationship between the point in question and three other non-collinear points is established, the coordinates of the point during motion can be calculated from the corresponding positions of the other three points.

The method is as follows:

Let B, C, D be the three measured points and A be the point in which translation is being calculated, and that the initial position of A which is A_o is known. Let us further assume in this case that $\overrightarrow{CB}$ is perpendicular to $\overrightarrow{CD}$. This method is general enough that it also applies without such restriction. However, this condition, as is shown later, simplified the calculation involved..

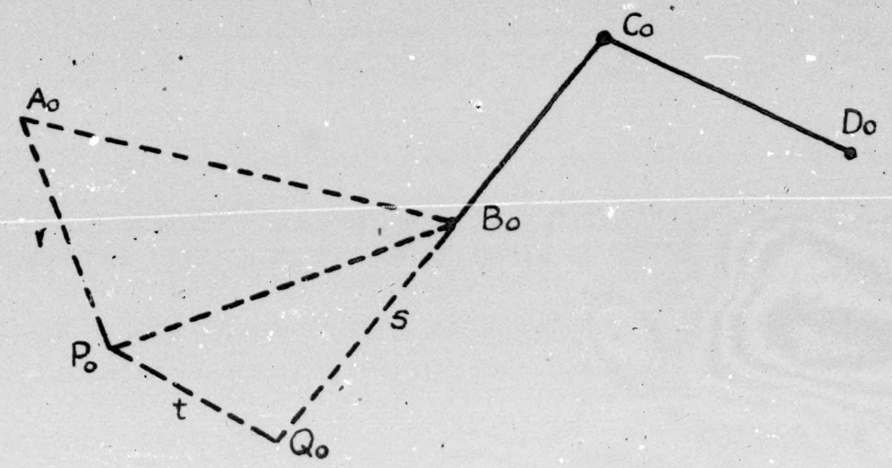


Fig. 3.

At the initial position,

$$\hat{\vec{n}}_o = \widehat{\overrightarrow{C_cB_o}} \times \widehat{\overrightarrow{C_oD_o}} \qquad \text{(Unit vector perpendicular to the plane BCD).}$$

$$\hat{\vec{n}}_o \cdot \overrightarrow{B_oA_o} = r$$

If we define P_o as the projection of point A on the plane $B_oC_oD_o$

then $\overrightarrow{P_oA_o} = r\,\hat{\vec{n}}_o$

$$\overrightarrow{B_oP_o} = \overrightarrow{B_oA_o} - \overrightarrow{P_oA_o}$$

On plane $B_oC_oD_o$,

$$\overrightarrow{B_oP_o} = \overrightarrow{B_oQ_o} + \overrightarrow{Q_oP_o}$$

$$\overrightarrow{B_oQ_o} = \overrightarrow{C_oB_o} \cdot (\overrightarrow{B_oP_o} \cdot \overrightarrow{C_oB_o})$$

$$\overrightarrow{B_oP_o} \cdot \hat{\overrightarrow{C_oB_o}} = s.$$

$$\hat{\overrightarrow{C_oD_o}} \cdot \overrightarrow{B_oP_o} = t.$$

For rigid body, r, s, t are constant.

At position 1.

$$\hat{\overrightarrow{n_1}} = \hat{\overrightarrow{C_1B_1}} \times \hat{\overrightarrow{C_1D_1}}$$

$$\overrightarrow{B_1Q_1} = \hat{\overrightarrow{C_1B_1}}\,(s)$$

$$\overrightarrow{Q_1P_1} = \hat{\overrightarrow{C_1D_1}}\,(t)$$

Then $\overrightarrow{B_1P_1} = \overrightarrow{B_1Q_1} + \overrightarrow{Q_1P_1}$

$$\overrightarrow{P_1A_1} = \hat{\overrightarrow{n_1}}(r)$$

$$\overrightarrow{B_1A_1} = \overrightarrow{P_1A_1} + \overrightarrow{B_1P_1}$$

From this, the position of A_1 is known.

Since the coordinates of points A_o and A_1 are known, the translation from A_o to A_1 is also known.

C) Rotational motion.

In rigid body rotation, the magnitude of rotation is the same for every point in the rigid body. Therefore, in this case we need only to calculate the rotation of one point in the rigid body.

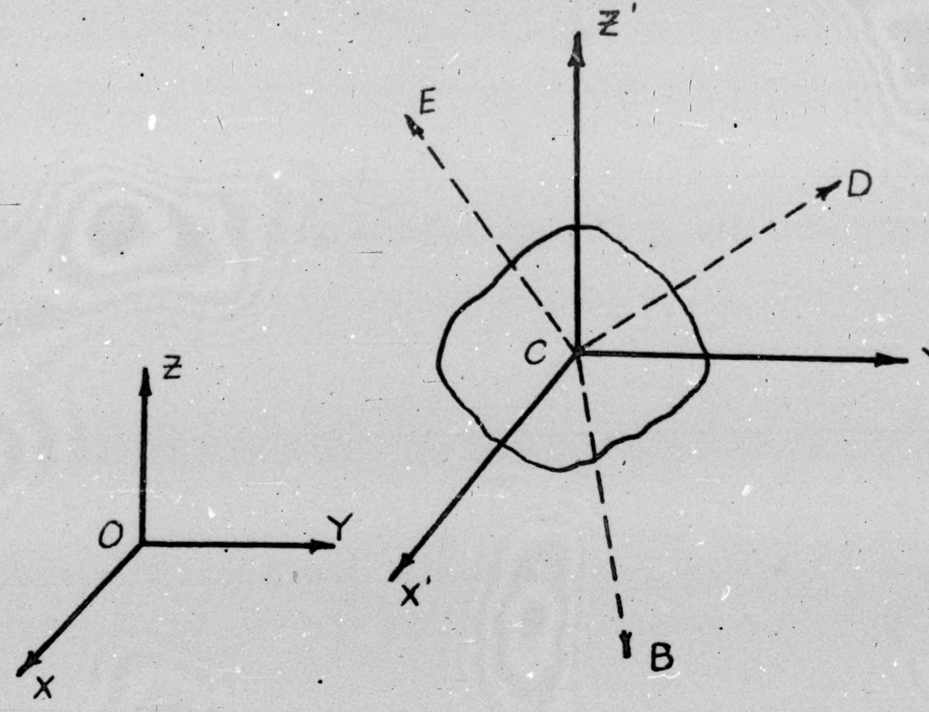


Fig. 4.

Let B, C, D be three points on a rigid body and that $\overrightarrow{CB}$ is perpendicular to $\overrightarrow{CD}$. Then we can say that point C is the center of a set of right-handed orthogonal axes with $\overrightarrow{CB}$, $\overrightarrow{CD}$ and $\overrightarrow{CB} \times \overrightarrow{CD} = \overrightarrow{CE}$ as the orthogonal vectors. This is a set of orthogonal axes which rotates with the body. Now, let CX', CY', CZ' be another set of right-handed orthogonal axes which is parallel to the reference axes OX, OY, OZ. Therefore CX', CY', CZ' form a set of fixed axes at point C which is always parallel to the X, Y, Z axes. In translation, the relative position of the two sets of orthogonal axes is unchanged. In rotation, the relative position of the two sets of orthogonal axes are changed, and the amount of change indicates the magnitude of rotation. The conventional method to describe the rotation of rigid body in space is the use of the three Euler Angles. For infinitesimal rotation, rotations can be represented by vectors. The following method involves the process to represent the three Euler Angles in vector forms for a subsequent calculation.

Euler Angle ψ, θ, ϕ.

Let C- X, Y, Z and C- B, D, E be two sets of right-handed orthogonal axes. The three Euler Angles represent the amount of rotation of BDE axes so that it would coincide with the XYZ axes.

Angle ψ

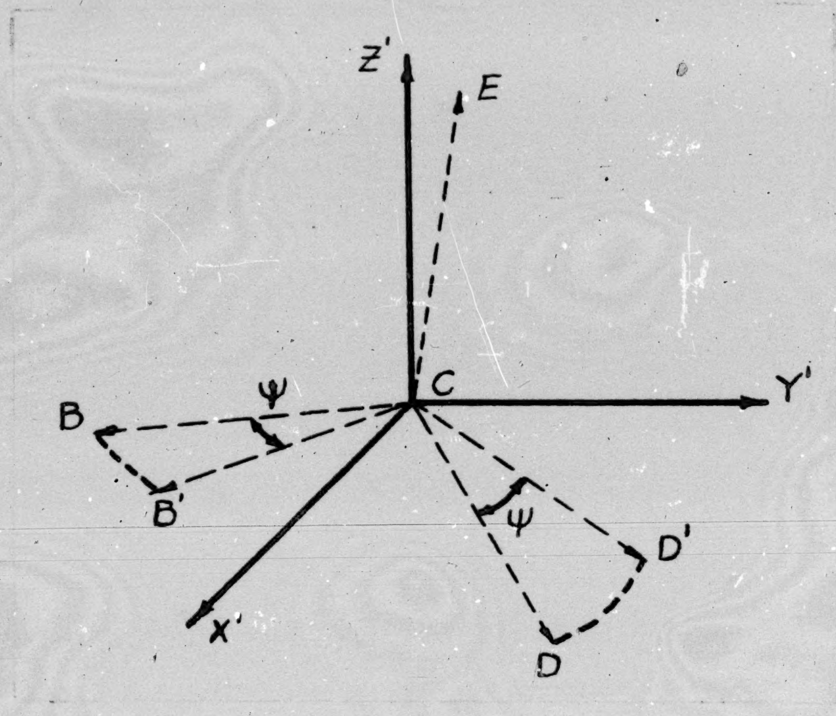


Fig. 5.

Rotate point D counterclockwise about the E- axis until it hits the Y' C Z' plane at D'. The angle subtended by CD and CD' is the angle . Point B at the same time will also be rotated to B'.

Angle θ

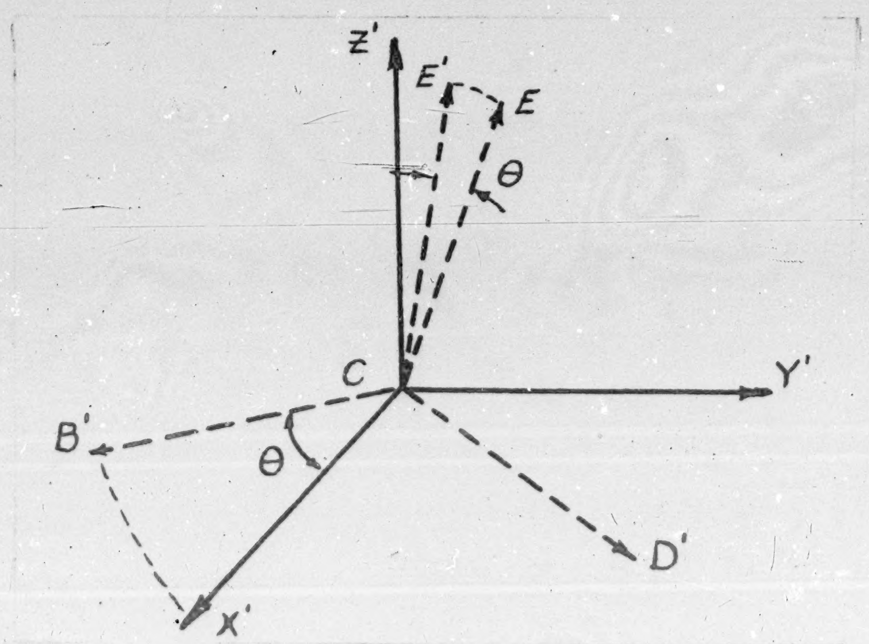


Fig. 6.

Now rotate point B' about the CD' axis until CB' coincides with CX. The angle subtended by CB' and CX is the angle . Point E will also be rotated to E' which is on the ZCY plane.

Angle ϕ

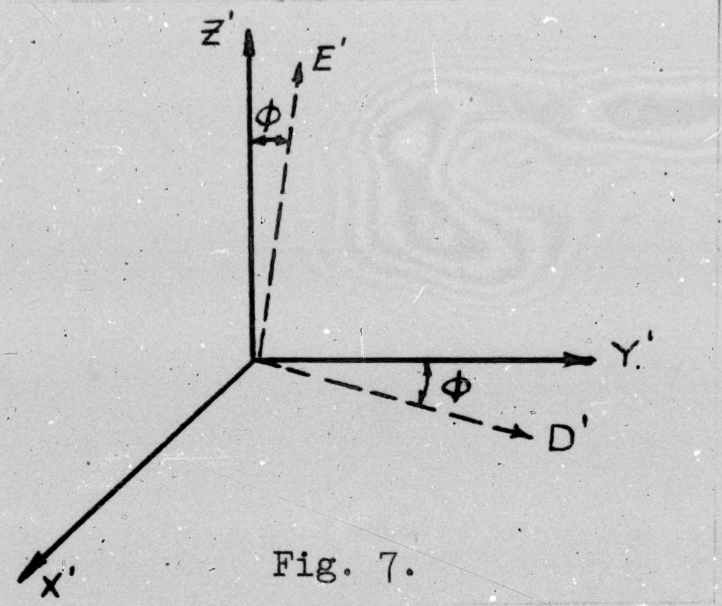


Fig. 7.

Rotate E' about the CX' axis until CE' coincides with CZ' axis. The angle subtended by CE' and CZ' is the angle . D' will also be rotated so that CD' coincides with CY' axis. Thus the two sets of orthogonal axes coincide.

Let W_x', W_y', W_z' respectively as the vector representation of magnitude of rotation around the X', Y', Z' axes. Then

1) Angle ϕ can be represented by W_x

2) Angle θ by the CD' axis.

$$W_z' = \theta\,(\hat{\overrightarrow{CD}}')\,(\vec{k})$$

$$W_y' = \theta\,(\hat{\overrightarrow{CD}}')\,(\vec{j})$$

3) Angle ψ by the CE axis.

$$W_x = \psi\,(\hat{\overrightarrow{CE}})\,(\vec{i})$$

$$W_y = \psi\,(\hat{\overrightarrow{CE}})\,(\vec{j})$$

$$W_z = \psi\,(\hat{\overrightarrow{CE}})\,(\vec{k})$$

$$W_x = \phi + \psi\,(\overrightarrow{\hat{CE}})\,(\overrightarrow{i})$$
$$W_y = \theta\,(\overrightarrow{\hat{CD}}')\,(\overrightarrow{j}) + \psi\,(\overrightarrow{\hat{CE}})\,(\overrightarrow{j})$$
$$W_z = \theta\,(\overrightarrow{\hat{CD}}')\,(\overrightarrow{k}) + \psi\,(\overrightarrow{\hat{CE}})\,(\overrightarrow{k})$$

Calculation of Rotation.

The previous section describes the three Euler angles and how they are represented. This section is concerned with the calculation of the magnitude of the three angles.

Referring to Figure 4, let the coordinates of point C be C_x, C_y, C_z. The equations of translation of axes are:

$$X = x' + h$$
$$Y = y' + k$$
$$Z = Z' + l$$

In this case, $h = C_x$, $k = C_y$, $l = C_z$.

$$B\,(x', y', z') = B\ \ (B_x - C_x), (B_y - C_y), (B_z - C_z)$$
$$D\,(x', y', z') = D\ \ (D_x - C_x), (D_y - C_y), (D_z - C_z)$$

Let us assume in this case that the order of the BDE axes according to the right-handed rule is B, D, E and that the rotation is such that CB would coincide with the positive CX' axes, CD with positive CY' and CE with positive CZ'. This, as we can see, is arbitrary because we are only interested in relative motion. However, it has to be consistent, that is, once we choose CD with OX', CD with CY' and CE and CZ', we have to use them for all calculations.

Let the direction cosine of

$\overrightarrow{CB}$ be b_1, b_2, b_3

$\overrightarrow{CD}$ be d_1, d_2, d_3

$\overrightarrow{CE}$ be e_1, e_2, e_3.

Calculation of ψ.

$$\hat{\vec{CE}} = \hat{\vec{CB}} \times \hat{\vec{CD}}$$

Equation of plane BCD, $e_1x' + e_2y' + e_3z' = 0$

Equation of plane Y'CZ', $x' = 0$

The intersection of the two planes is a straight line $e_2y' + e_3z' = 0$. As is shown previously the point D' lies on this line, therefore $\vec{CD'}$ is the unit vector of $\vec{j} - \frac{e_2}{e_3}\vec{k}$.

Since is defined as the angle between $\vec{CD}'$ and $\vec{CD}$

$$\cos\psi = \frac{(d_1\vec{i} + d_2\vec{j} + d_3\vec{k})}{\sqrt{d_1^2 + d_2^2 + d_3^2}} \frac{(\vec{j} - \frac{e_2}{e_3}\vec{k})}{\sqrt{1 + (\frac{e_2}{e_3})^2}}$$

Calculation of θ.

$$\hat{\vec{CB}}' = \hat{\vec{CD}}' \times \hat{\vec{CE}}$$

Angle θ is the angle subtended by $\vec{CB}'$ and $\vec{CX}'$.

$$\cos\theta = \hat{\vec{CB}}' \cdot \vec{i}$$

Calculation of ϕ.

$$\cos\phi = \hat{\vec{CD}}' (\vec{j})$$

At position 0, Wx'_o, Wy'_o, Wz'_o are calculated, and similarly at position 1, Wx'_1, Wy'_1, Wz'_1 are calculated. The difference between the two is the amount of rotation about the XYΣ axes from position 0 to position 1.

Concerning the magnitude of , , .

The previous paragraph shows that we only calculate the cosine of the angles. If the angle varies from 0 to 2π, then for each cosine, there are two possible values of the angle. In order to avoid confusion, the following convention is adopted to identify the magnitude of the angle.

For ψ

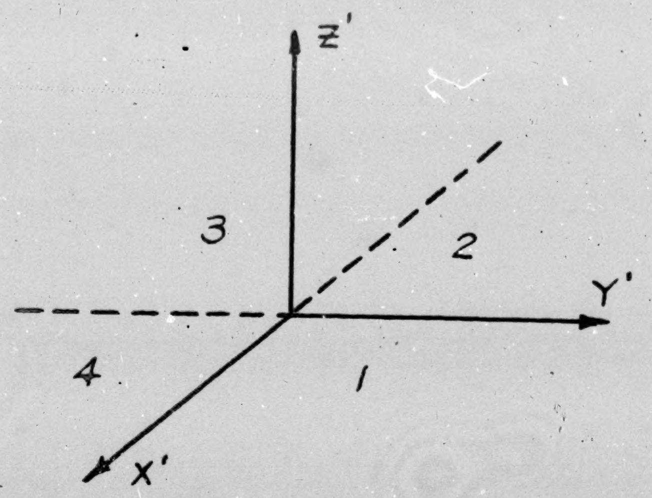


Fig. 8

Since in this case the sign of Z' does not enter, therefore in

Quadrant	1	X' +	Y' +
"	2	X' -	Y' +
"	3	X' -	Y' -
"	4	X' +	Y' -

We can identify CD in any quadrant by noting the signs of X' and Y'

ψ = /arc cos ψ/ if D is in 1st Quadrant

= 2π -/arc cos ψ/ " " " " 2nd "

= π +/arc cos ψ/ " " " " 3rd "

= π -/arc cos ψ/ " " " " 4th "

For θ.

Angle θ is the angle between CB' and + X'-axis

θ = /arc cos θ/ if Z' component of CB' +

=-/arc cos θ/ " " " " " -

For angle ϕ.

$$\phi = /\text{arc} \cos \phi/ \qquad \text{if } Z' \text{ component of } CD' \ -$$

$$= -/\text{arc} \cos \phi/ \qquad '' \quad '' \quad '' \quad '' \quad '' \ +.$$

The justification for using this convention rather than that of turning counter clockwise all the time is that we know CD' is very near the ± Y'-axis. Therefore using the convention of turning in one direction would add confusion to the magnitude of when CD' at + Y'-axis changes from + Z' to - Z'.

D) The Orthogonal reference axes.

As was stated previously, a set of fixed orthogonal axes is needed so that all motions can be referred in order that the motion of one foot can be compared with that of the other foot. The new axes so chosen have to be identical for both cases. No attempt is being made to justify that the present method for locating the new axes is absolute so that each and everyone is similar. However, it is believed that this is the best available method.

The three orthogonal axes so chosen are:

1) The line joining the center of the two ends of the tibia.
2) The line joining the geometrical centers of the medial and of the lateral tibial condyle.
3) The third axes is parallel to the horizontal plane and orthogonal to the above two axes.

The location of the Reference Axes.

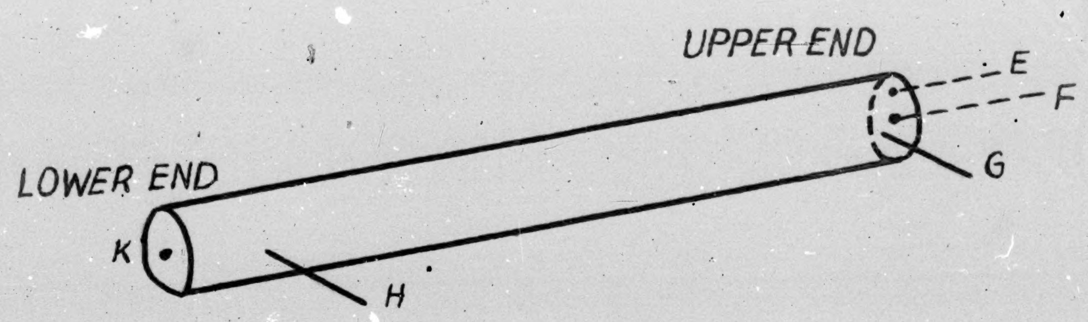


Figure 9. Tibia.

Referring to Figure 9, let the points F and K be the centers of the two ends of the tibia, and FKHG be the horizontal plane. The point E will be on the vertical plane EFK which is perpendicular to the plane FKHG. The points G and H are such that the line GH is parallel to the line FK. The coordinates of points G, H, F, E are known. Since line GH is parallel to line FK, therefore $\overrightarrow{GH} = \overrightarrow{FK}$.

Figure 10.

Referring to Figure 10, $\overrightarrow{FE}$ is not necessarily perpendicular to $\overrightarrow{FK}$.

However, $\overrightarrow{FE} \cdot \hat{\overrightarrow{FK}}$ = distance (LF)

$\hat{\overrightarrow{FK}}$ (distance LF) = $(Lx - Fx)\,\overrightarrow{i} + (Ly - Fy)\,\overrightarrow{j} + (Lz - Fz)\,\overrightarrow{k}$

Then L (x, y, z) is known,

$\hat{\overrightarrow{LE}}$ is known.

The third orthogonal vector is $\hat{\overrightarrow{LE}} \times \hat{\overrightarrow{FK}}$. Let new x"-axis be on FK

$\hat{\overrightarrow{FK}}$ is on new x"-axis toward positive direction.

The Z"-axis is parallel to $\hat{\overrightarrow{LE}}$

The Y"-axis is parallel to $\hat{\overrightarrow{LE}} \times \hat{\overrightarrow{FK}}$.

Thus these are the new set of orthogonal axes.

If we choose the origin of the axes as the upper end face of the tibia,

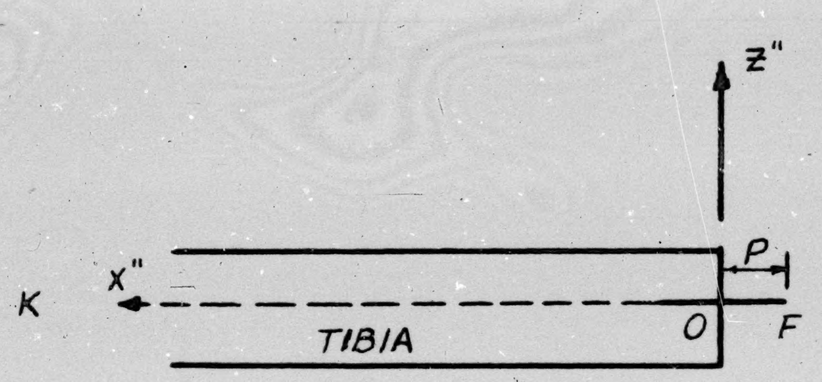


Figure 11.

$$\text{Let } \hat{\overrightarrow{FK}} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$$

$$\hat{\overrightarrow{LE}} = c_1 \vec{i} + c_2 \vec{j} + c_3 \vec{k}$$

$$\hat{\overrightarrow{LE}} \times \hat{\overrightarrow{FK}} = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}$$

Then $Ox'' = Fx - a_1(p)$

$Oy = Fy - a_2(p)$

$Oz = Fz - a_3(p)$

The point $O(x, y, z)$ is chosen as the origin of the new set of orthogonal axes.

<u>Translation and Rotation of Axes.</u>

Translation of axes to new origin.

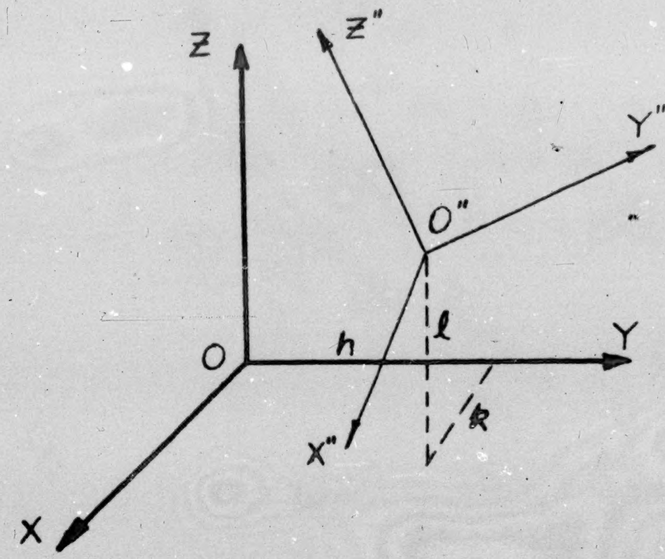


Figure 12.

If all measurements are recorded in x, y, z coordinates, the equations for translation of axes are

$x' = x - h$

$y' = y - k$

$z' = z - l.$

Rotation of the axes.

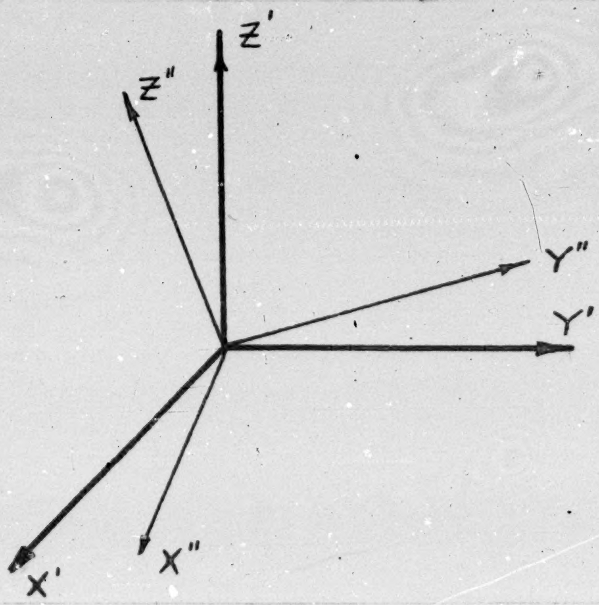


Figure 13.

Let a_1, b_1, c_1,; a_2, b_2, c_3; $a_3b_3c_3$ be the direction cosines

of the three orthogonal axes OX", OY", and OZ" with respect to the orthogonal axes OX', OY', and OZ'. Then the equations for rotating the axes into the position O-X"Y"Z" are:

$X'' = a_1x' + b_1y' + c_1z'$

$Y'' = a_2x' + b_2y' + c_2z''$

$Z'' = a_3x' + b_3y' + c_3z''$

Applying these equations to this case, we have

$Ox = h$

$Oy = k$

$Oz = l.$

Translation of axes to new origin O

$X' = x - h$

$Y' = y - k$

$Z' = \ - l$

Rotation of the axes at O.

Let the direction cosines of

x'-axis be a_1, b_1, c_1

y'-axis be a_2, b_2, c_2

z'-axis be a_3, b_3, c_3

Then the equations for rotation are:

$x'' = a_1x' + b_1y' + c_1z'$

$y'' = a_2x' + b_2y' + c_2z'$

$z'' = a_3x' + b_3y' + c_3z'$

Chapter 4
Experimental Procedure

A) Introduction.

Mr. Rothstein in 1956 had demonstrated the feasibility of using the two-camera system for the quantitative study of the motion of the foot. It has been shown subsequently that a stereoplotter could be used to create a stereo-model from the diapositives of the metric photographs and a scanning system to obtain information about spatial coordinates of any discrete points from the stereo-model. However, a scanning system can perform two functions. Firstly, information with regard to discrete spatial coordinates can be obtained from a stereo-model, and secondly, similar information can also be obtained from the original model. The direct measurements of coordinates of points from the original model have certain advantages. First of all, it eliminates all the intermediate processes, such as the use of special cameras, metric photographs, diapositives and a stereoplotter. This direct-measurement method also involves lesser expenditure since the equipment is already available in the M.I.T. Photogrammetry Laboratory. However, direct measurement by this technique is limited to the measurement of stationary objects and hence can be used only on specimens and not on living feet during function. Since we are interested, at present, only in the types and ranges of motion of different joints, direct-measurement method seems to be the best available technique.

The present experimental procedure involves the following principle steps:

1) The scanning system to obtain information of the coordinates of discrete points on the actual model.

2) The readont system to transmit the acquired information into punch cards so that the computer can do the calculation.

3) The calculation which involves the use of the IBM 704 to compute the translation and rotation, and to do other computation as well if desired.

The first experiment was performed in January 1958, and since then 18 amputated limbs have been tested. Due to the complexity of the system as a whole, certain unforseen errors were discovered in the course of the experiment and during the preliminary calculation. Changes, therefore, were made. It is felt that the data recorded from experiment 10 onward are more reliable than those from the preceeding experiments.

The numerical calculations involved in the determination of joint movement are so long and tedious that the method would be impractical unless automatic methods are developed. The need for automatic methods was made abundantly evident in the work at the University of California. For this reason, a computer program is being written for these calculations and should be in use sometime in June 1958.

B) Scanning System.

The function of the scanning system is to translate the measuring mark through the model along a system of xyz instrument

axes. The scanning unit which is presently being used on the experimental M.I.T. system is the Nistri Electro-Coordinatometer which is a commercial unit. A noteworthy distinction of this unit is that all transmission of motions between components of the unit is electrical instead of mechanical. This provides considerable flexibility in the use of the unit for different applications.

A functional description of the unit and its associated components is as follows:

a) The basic scanning unit consists of three lead screws, the rotation of which translates the platen carrying the measuring mark through the model in x, y and z directions.

b) Each lead screw is driven by an individual synchronous motor which in turn is controlled by a master synchro. Two of the master synchros are on hand wheels and the third is on a foot wheel. These make possible the manual control of the translation by the operator. The foot wheel also has a constant speed rheostat motor drive pedal for continuous motion through the model along one lead-screw axis.

c) The control wheels are interchangeable with regard to the scanning motion controlled by interchanging leads at a master bus box.

d) The speed ratio between the control wheels and the lead screws can be varied over a wide range by variable gearing.

e) An analog counter is attached to the Z axis with variable gearing for selecting the Z scale. Metric scales are available along the x and y machine axes, with fine adjustment to .01 m.m.

Measuring Device.

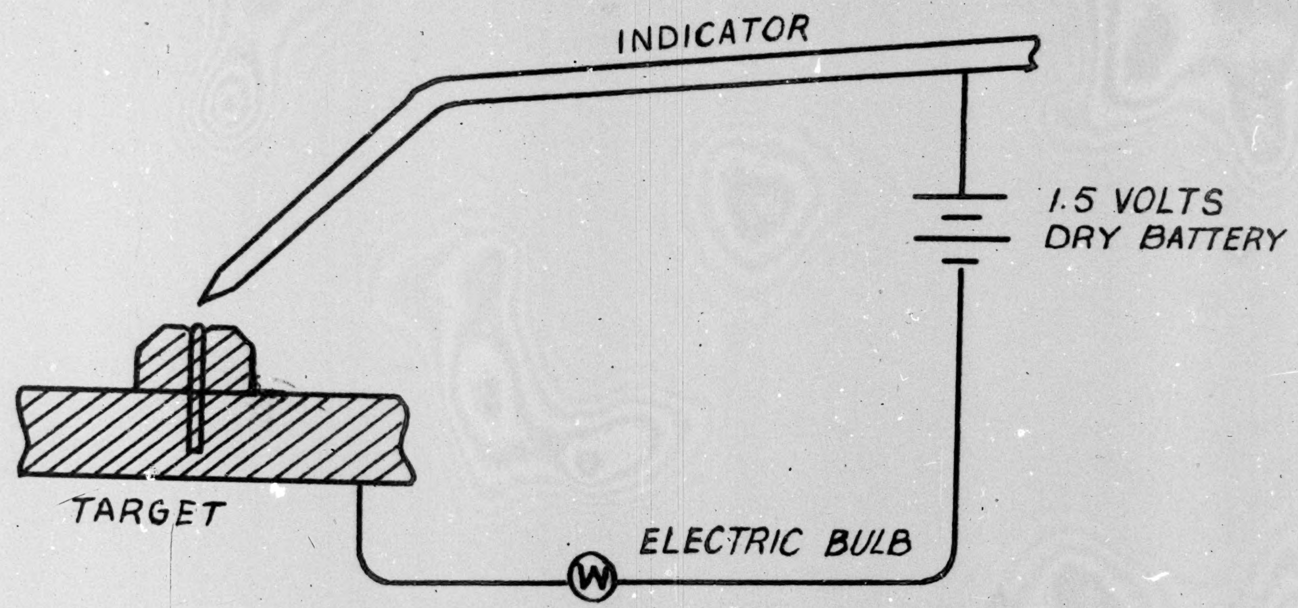


Fig 14. Measuring Device.

The measuring device in this case consists of a pointed thin steel rod of 1/8" diameter which is clamped to a vertical spindle by an adjustable clamping device. The spindle in turn is mounted vertically on the rider through an elongated slot. The adjustable clamping device enables the indicator to have three degrees of freedom which are essential in the sense that the pointer has to make contact with whatever point is to be measured. The coordinates of the target point are recorded when the pointer of the indicator is in actual contact with the target point. Since the target point is actually the cross-sectional area of a copper wire of .5 m.m. diameter, it takes considerable effort each time to ensure that the point of the indicator is in contact with the surface of the target. To facilitate this, an electric circuit with one 1-1/2 volt battery and a flash-light bulb is connected in series with one end to the target and the other end to the measuring device. This is an open circuit ordinarily, and it becomes a closed circuit only when the pointer of the indicator comes in contact with the target point. Thus the lightening of the flash-light bulb indicates actual contact. This device not only reduces the effort and concentration on the part of the operator but at the same time reduces the time of operation considerably.

C) Read out system.

The function of the entire readont system in the M.I.T. Photogrammetry Laboratory is to record and store the x, y, z coordinates

of a position. This automatic read-out system can in general be divided into two parts:

1) Encoding system

2) Storage system.

Encoding System.

The encoding system developed is primarily designed for the purpose of encoding x, y and z coordinates cooresponding to the position in the form of electrical pulses. A description of the Incremental Feedback Pulse Encoder as used in the readont system is given on page 22 of Publication 109 of the M.I T. Photogrammetry Laboratory. This type of counting system is presently being installed to record x, y and z motions made by the measuring mark fixed on the rider of the scanning system. The rotations of the lead screws are transmitted to the reading heads of the encoder by means of three synchronous motors, and also through a variable gearing which permits readings of the encoded x, y and z distances to be made at various scales. In this case the model is the actual specimen and therefore the scale in x, y is one to one; that is the actual scale, however, in the z-direction the scale varies from 1 = 1.3 to 1 = 10 and corresponding correction has to be made for each scale. The smallest encoded increment varies from .025 m.m. to .125 m.m.

It is found that a very favorable arrangement relative to effective gear backlash, effective inertia of the system, and driving torques required from the handwheel driven sychronous motor system is obtained if the synchronous motors are operated at a speed 4 times that of the lead screws. The error due to the synchronous lag is well

within a hundreth of a millimeter. However, as is shown in a later section, the synchronous lag error is comparatively much smaller than the error due to other causes, therefore, it was decided to obtain the largest speed possible so as to speed up the experiment. The present synchronous motors are operating at a speed 4/3 times that of the lead screws.

This is a counting type encoder, and the maximum counting speed of the system is approximately 1000 counts per second. The capacity of the x, y and z counters is flexible since each digit counter permits as many additional digits to be added as desired.

Storage System

Information is read out of the counters to permanent storage only when the shafts are stopped. This is done by transferring the count on each digit in succession to a separate counter tube, which is then read out through ten thyratrons and ten relays to the associated output equipment.

Programming of the digits is accomplished by a stopper switch. The x, yz coordinates of each position are recorded separately, and the first four digits of each card are for identification of information associated with the experiment. Hence, in each experiment there are a maximum of four hundred positions that can be identified independently.

The readout system being used on the experimental system includes a remotely driven electric typewriter, a serial entry card punch, and a tape punch. All three devices are being driven in

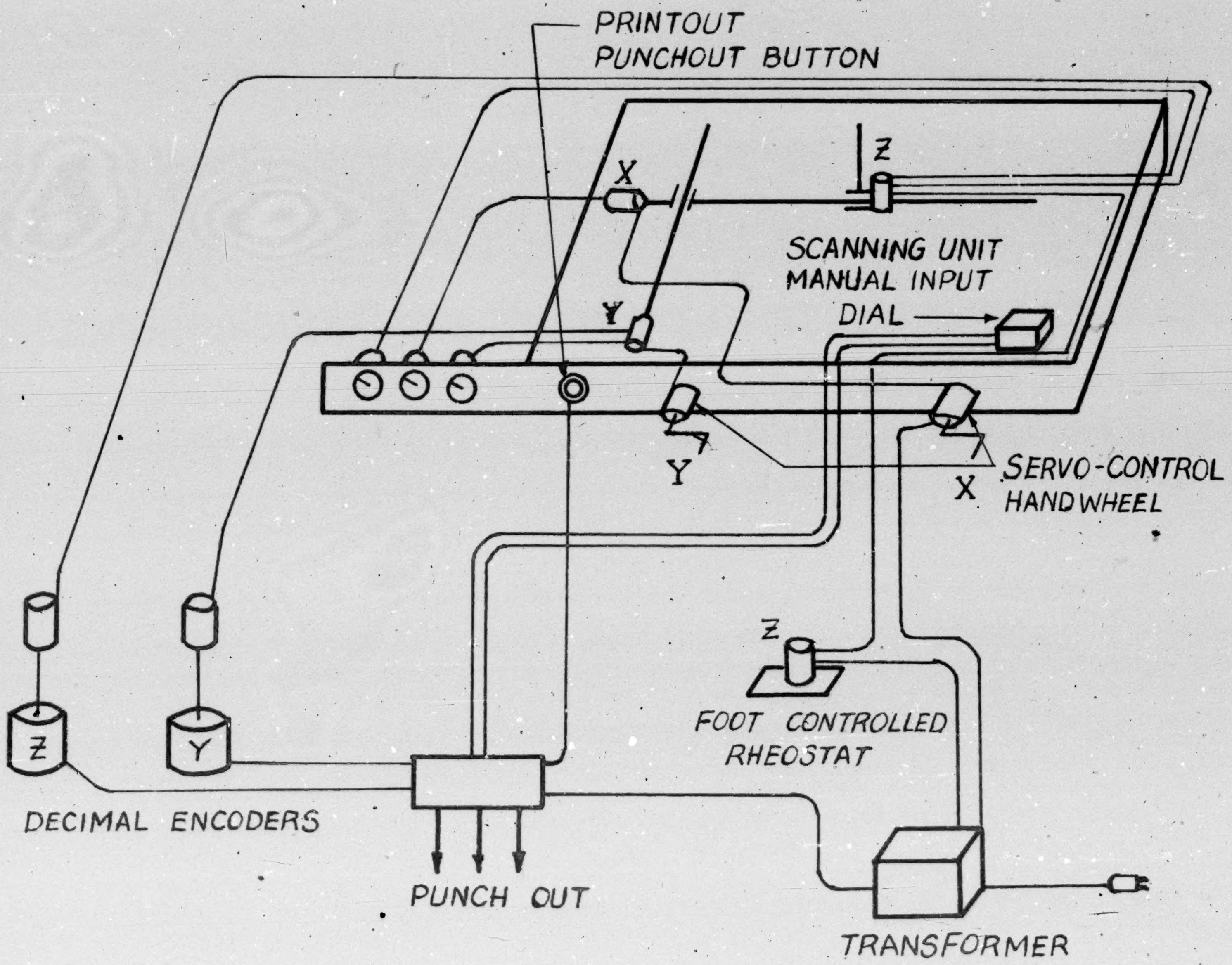


Fig. 15. Scanning and Automatic Readout System.

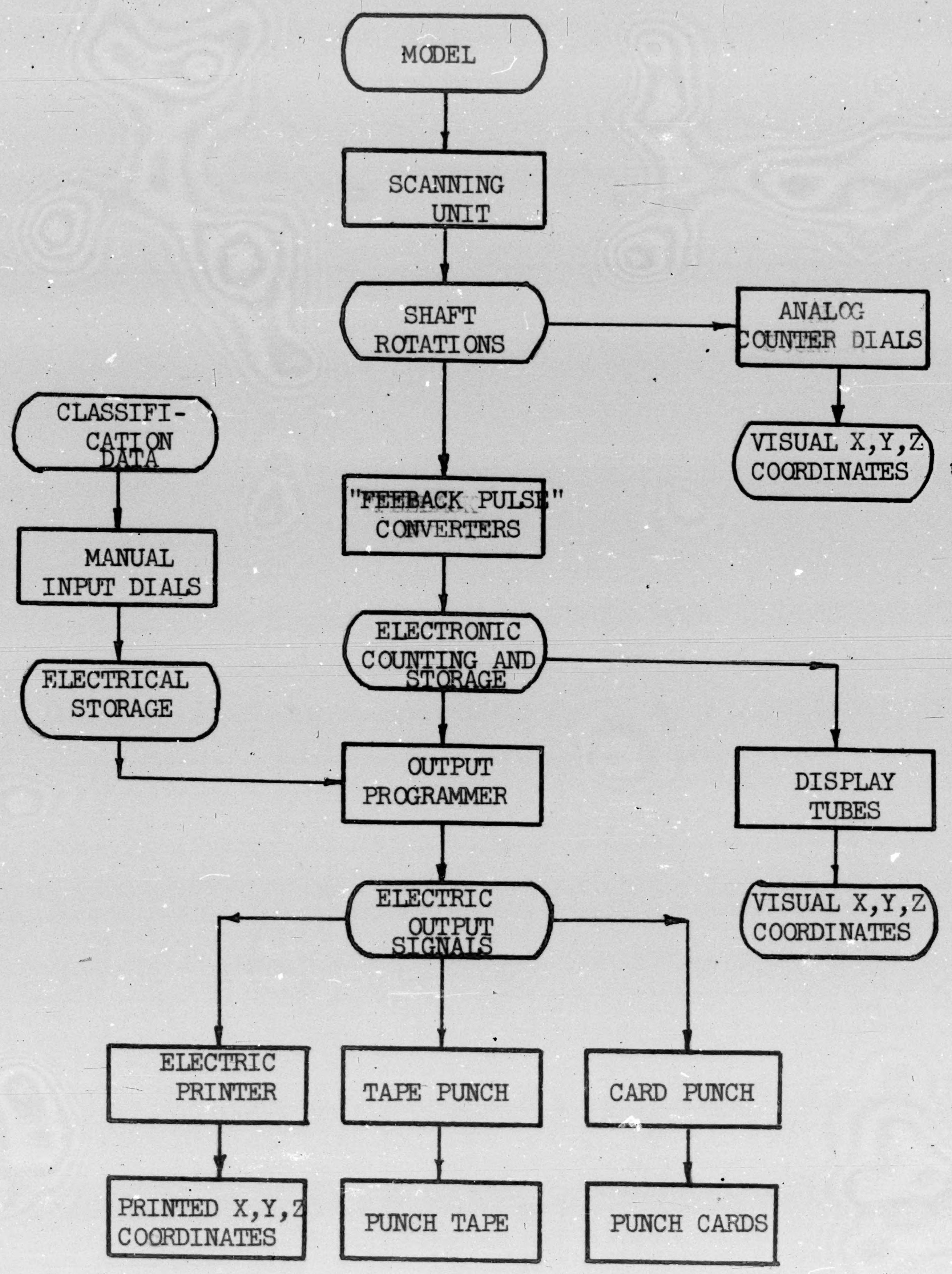


Fig. 16. Instrumentation Components and Data Forms.

parallel from the step-switch scanners. Coding instructions for handling the input data in two different computers, one card programmed (IBM 650 or IBM 704) and the other tape programmed (Bendix 615D) are also included automatically at their proper places by the two step scanners. If only the card punch and typewriter are being used in this system, it would be feasible to use an internally programmed serial entry card punch (IBM 526 Summary Punch) in place of one of the stepper switches. If the input instructions to the computer were not included in the outputs at this point, the second stepping switch could also be dispensed with.

D) The Experiments.

It seems appropriate at this stage to briefly describe the experiments which have been carried out in the Department of Civil Engineering at M.I.T. since January, 1958. Changes have been made and will be made whenever it seems desirable, and therefore the description of the experiments represents that of the present work.

The experiment in general can be divided into three steps:

1) Preparation of specimens
2) Actual experiment
3) X-ray of the specimen for identification.

The first and third steps are normally carried out in the Massachusetts General Hospital and the second step is carried out at M.I.T.

1) Preparation of specimens.

Lower extremities, amputated above the knee because of

arterial insufficiency, malignancies etc., were prepared by removing the skin, subcutaneous tissues and muscle masses, leaving the ligamentous structures intact. The limbs so prepared were then sealed in plastic bags and preserved by freezing.

Immediately prior to an experiment the limb was thawed and the following markers were inserted for the establishment of the reference axes.

1) A small wire brad was driven into the geometrical centers of the medial and of the lateral tibial condyles. The geometrical centers were located by measurement with a caliper. The location of these geometrical centers for these preliminary experiments was therefore only approximate. The geometrical center of the medial condyle is the point E as described in the Theory.

2) The mid-point between the geometrical centers of the tibial condyles was also marked by a wire brad. This is the point F in the Theory.

3) Similar markers were inserted at the level of the tibial articular surface of the ankle midway between the anterior and posterior margins of the medial and lateral malleoli.

4) Two markers were inserted in the tibia, one at the level of the tibial tubercle, the other in the supra malleolar region. These were inserted by means of a drill press in a postereo-antereo direction with the limb clamped in a bracket in such a position that the plane of the markers in the tibial condyles was perpendicular to the direction of the drill press. The tibial tubercle and supra malleolar markers were placed on a line laid off on the posterior aspect of the tibia

between a point midway between the superfiscial surfaces of the two malleoli at the ankle and the mid-point between the two markers in the tibial condyles at the knee. The points of the two markers sticking out of the anterior surface of the tibia at the tibial tubercle and the supra malleolar region are the points G and H as denoted in the Theory.

In order to mount the specimen firmly, two 3/16" threaded rods were drilled through the tibia in the same plane as the aforementioned supra malleolar and tibia-turbucle markers. The threaded rods were locked by nuts turned up snugly against both surfaces of the tibia. Subsequently when the specimen was mounted on the stand, these threaded rods were clamped securely to the four uprights of the mounting stand. (See Fig. 19).

Handles with which to move and fix the talus and calcaneus were provided by inserting 1/4" threaded rods as follows:

1) In the talus the handle was inserted in a posterior-anterior direction just above the articular surface of the posterior facet of the subastragalar joint. This position was found to be most satisfactory because the rod would not impinge against either the tibia or calcaneus in any position of the foot or ankle.

2) In the calcaneus, the handle was inserted in an anterior-posterior direction through the tubercle. In this way the handle for this bone was nearly parallel to the sole of the foot and would not interfere with movements of the handle placed in the talus.

The targets used to record the movements of the tibia, talus

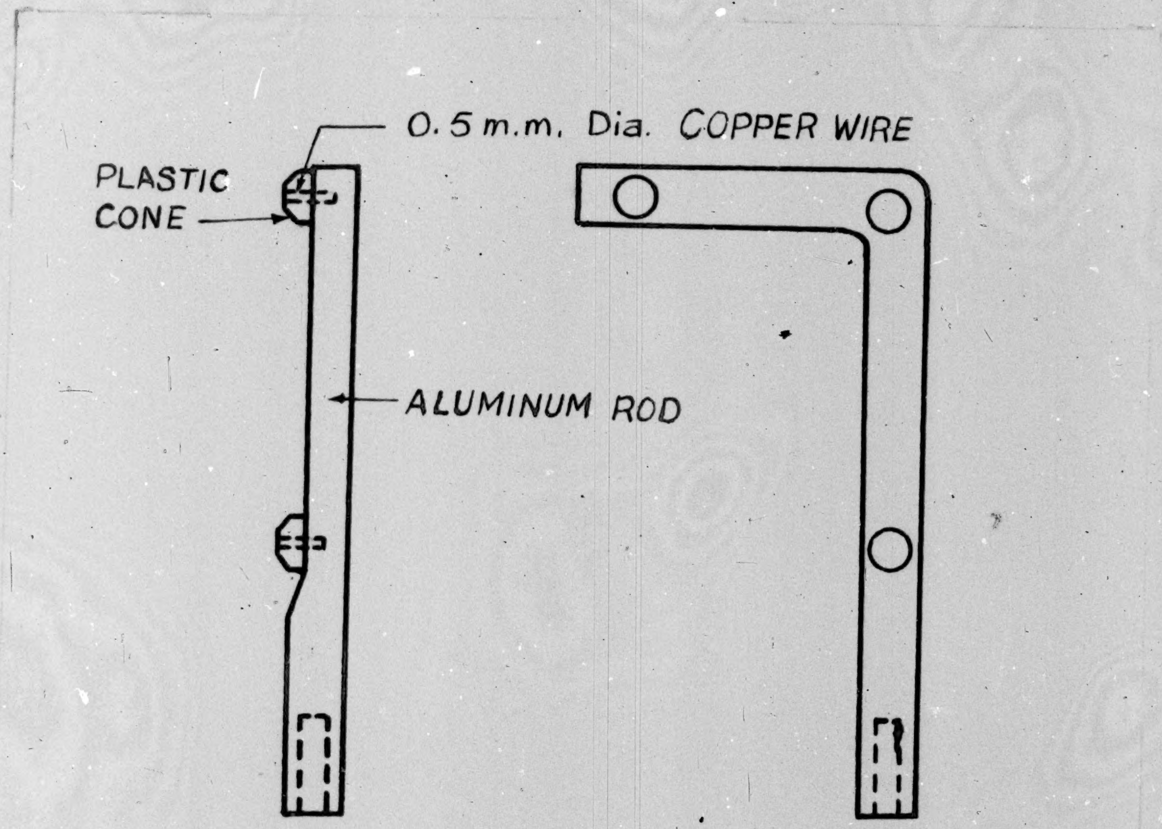
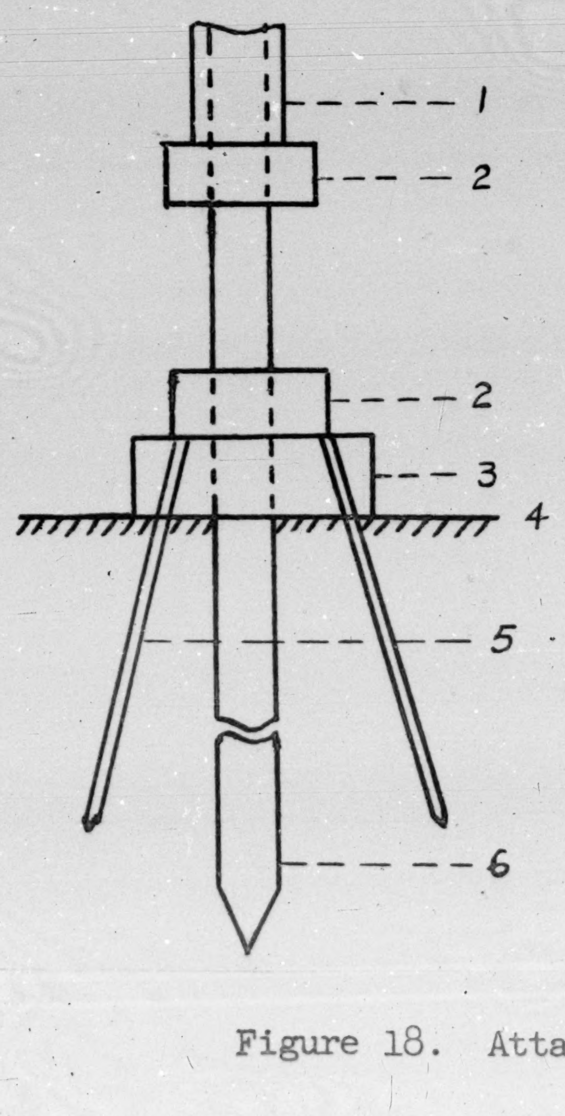


Fig. 17. Target.

and calcaneus are L-shaped aluminum rods to which are affixed three target points. These points consist of copper wires 0.5 m.m. in diameter projecting through a plastic cone cemented to the aluminum rod. The three target points are so located that two lines joining them together with an imaginary third line perpendicular to them are orthogonal to each other in accordance with the assumption discussed in the section on Theory. Each target is fixed to the appropriate bone by means of a threaded 3/16" aluminum rod. The target is threaded onto the rod and held by a lock washer. The rod, in turn, is threaded onto the bone and is held in place against rotation or displacement by an aluminum plate fixed to the bone surface, two oblique pins and a lock nut. (See Fig. 18).



1. Target
2. Lock nut
3. Aluminum plate
4. Bone surface
5. Oblique pin
6. Rod fixed to bone

Figure 18. Attachment of Rod to Bone.

Placement of the targets in order to allow free movement and yet keep the targets as close together as possible at all times in order to expedite the recording of the data proved difficult at first. Satisfactory position as shown in Fig. 20 was ultimately achieved by angulating the threaded rods and inserting them on the antero-lateral aspects of the talus and calcaneus.

2) The Actual Experiment.

The experiment is performed at the photogrammetry laboratory of M.I.T. The tibia is fixed to a stand and the bones to be studied are moved by moving the handle. The positions of the target points are measured by the scanning system and recorded. At present the study is confined to the movements of the ankle and subastragular joints, that is the movements of the talus, calcaneus and fibula. The procedure employed is to place the foot in various positions from maximum plantar flexion to maximum dorsiflexion. At each of these positions of plantar and dorsiflexion the talus is rotated internally and externally. At each position of external and internal rotation of the talus, inversion and eversion of the calcaneus is determined. The positions of the talus, calcaneus and fibula are thus determined in each instance by recording the x, y, z coordinates of the appropriate targets at each position in which the foot is placed.

The stand where the leg is being held during the experiment consists of a steel block measuring 12" x 4" x 1-1/2" which is adhered to a steel plate of 26" x 16" x 1/6" by pressure sensitive

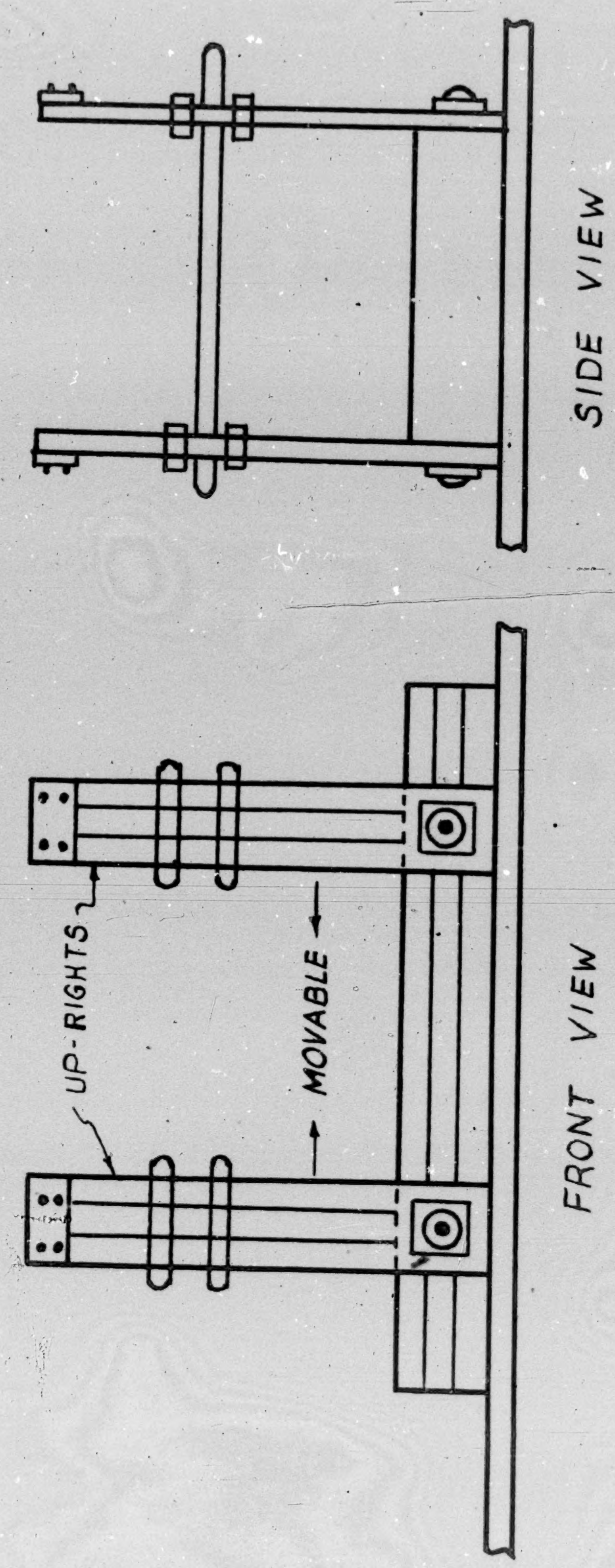
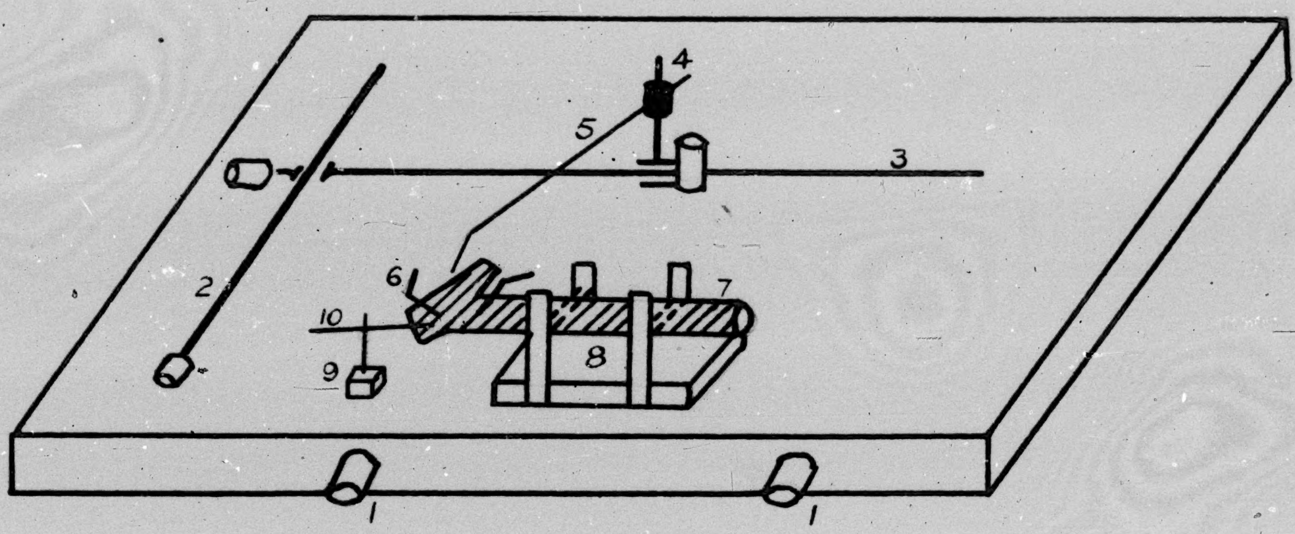


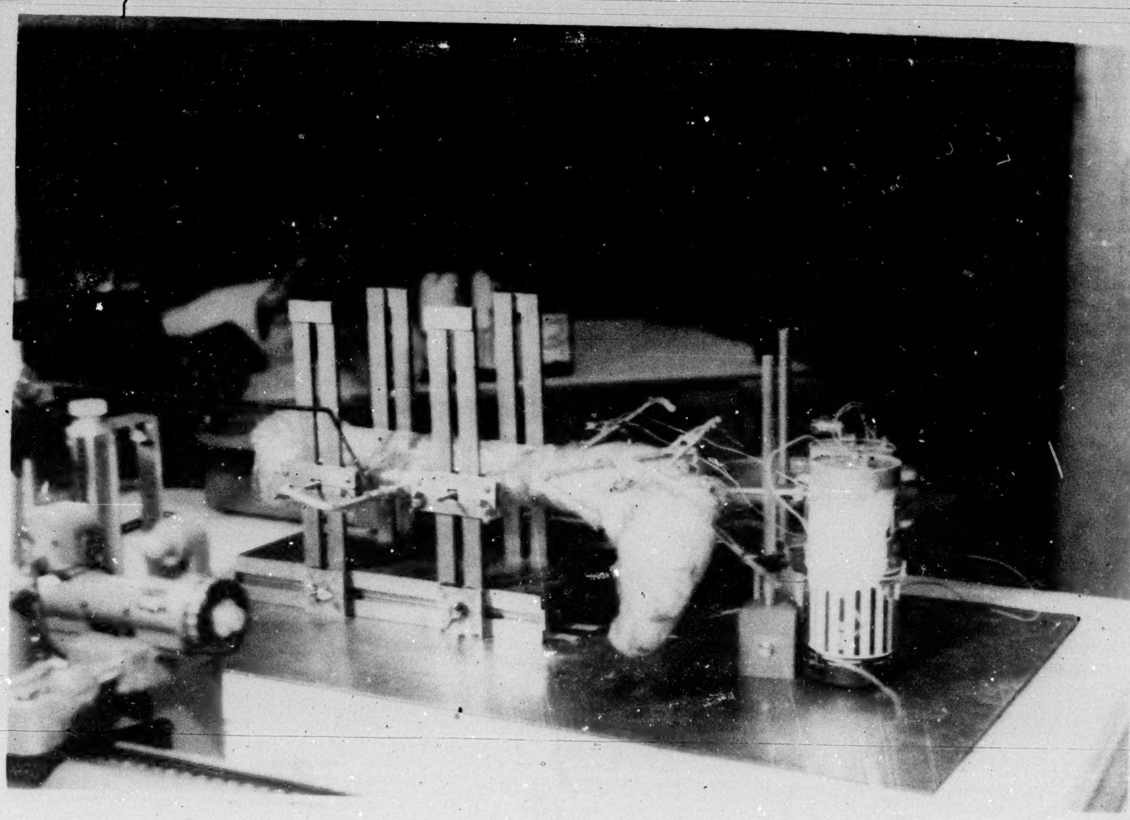
Fig. 19. Mounting Device.



1. Servo-control hand wheel
2. X-axis
3. Y-axis   Scanning System
4. Z-axis
5. Indicator
6. Target
7. Specimen
8. Mounting Device
9. Magnicator
10. Handle

Fig. 20. Scanning System and Measuring Device.

Top View



Front View

Fig. 21. Specimen with Experimental Set-up.

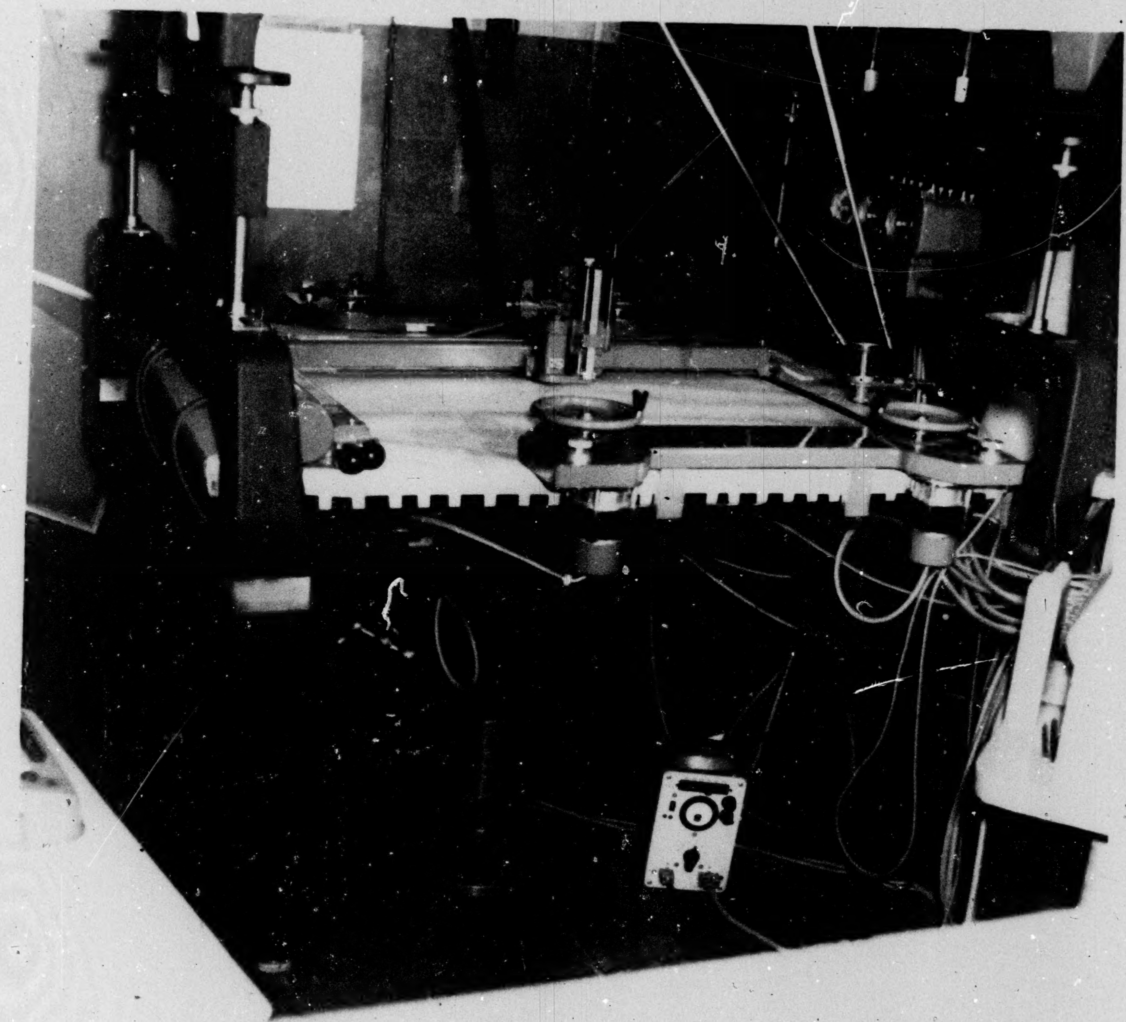


Fig. 22. View of Scanning System Showing Control Elements

tape. There are two up-rights with elongated slots which are attached to each side of the steel block through a T-slot with wing-nuts for setting so that horizontal adjustment is permissible. On each of the up-rights there are V-shaped clamps on top and bottom with wing-nuts for quick setting to allow for vertical adjustment. The two steel rods through the tibia are clamped to the up-rights by the V shaped clamps so that the tibia is held rigidly in position.

At each position where measurements of target points are being made, the handles in the talus and calcaneus are held in position by the magnicators which are permanent magnet-type magnetic chucks made by the Brown and Sharpe Company. The operator of the scanning system uses the two hand-wheels and the foot-pedal, each controlling one coordinate, to guide the measuring mark to contact the target points. At present, the automatic readout system is not completed yet, and manual recording of the coordinates is being done. This not only slows the experiment, but also creates opportunity for errors in recording.

3) X-Ray of the Specimen for Identification.

After completion of the measurements, x-rays are taken with a tube-to-target distance in excess of 7 feet to minimize distortion. The x-ray projections obtained may be described as follows:

1) Routine anterior-posterior projections of the whole specimen.

2) After disarticulation of the tibia, fibula, talus and calcaneus, two projections of each bone with the targets still in place. Each bone is held in a wooden clamp which in turn is attached to an adjustable camera mount. By this device the talus, calcaneus and

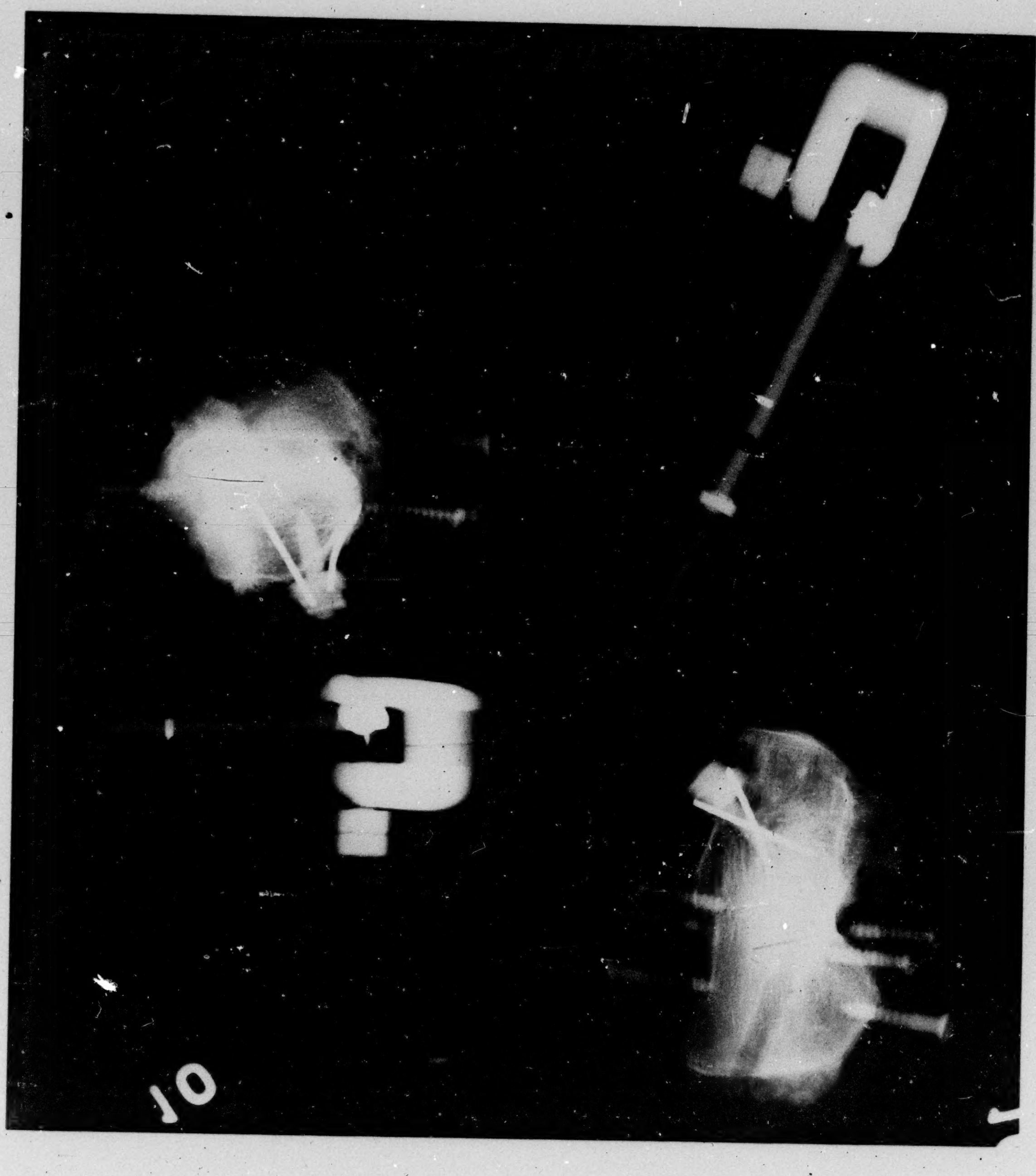


Fig. 23. Reproduction of the Roentgenogram of the Calcaneus.

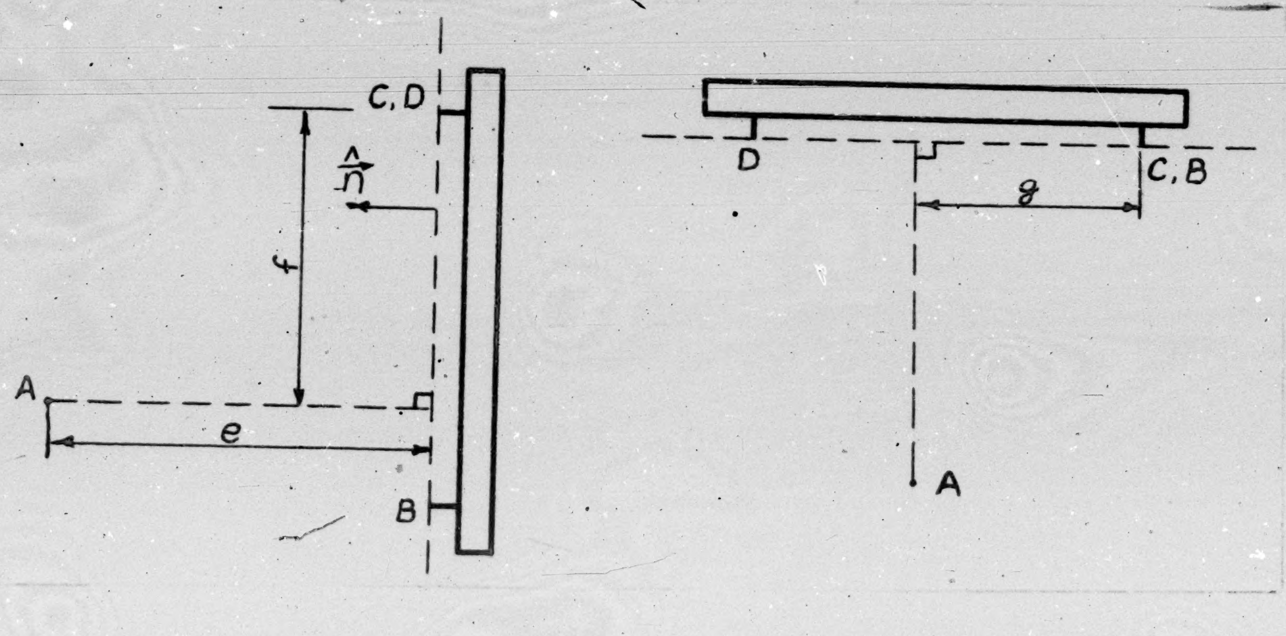
fibula are placed with their targets in the following positions:

(a) Points C and D on a plumb line

(b) Points C and B on a plumb line.

Accurate positioning so that points C and D, and points C and B are in fact superimposed in the x-ray pictures proved difficult as shown in the x-ray picture of the calcaneus reproduced in Fig. 23. With these x-rays, it is possible to determine the relationships of these target points to any point on the bone, as for example the geometric center.

The following is a description of the method:



Picture 1. Picture 2.

Figure 24. Orientation of Target.

Let the three points on a target be B, C, D and that point A be any point in the bone. The first x-ray picture is taken when

the target is oriented into a position when points C and D coincide. The second picture is taken when the points C and B coincide. Therefore in both pictures, the points C, D and B will form a straight line. Referring to Figure 24, let the unit vector

$$\hat{\vec{n}} = \hat{\vec{CB}} \times \hat{\vec{CD}}$$
$$\hat{\vec{m}} = \hat{\vec{CB}}$$
$$\hat{\vec{O}} = \hat{\vec{CD}}$$

Furthermore, let the perpendicular distance from point A to plane BCD be e, the perpendicular distance to line CD parallel to plane BCD be f, and the perpendicular distance to line BD parallel to plane BCD be g. Then because of the fact that the target is made in such a way that $\vec{CD}$ and $\vec{CB}$ are orthogonal to each other, it follows:

$$\vec{CA} = \hat{\vec{n}}\,(e) + \hat{\vec{m}}\,(f) + \hat{\vec{O}}\,(g)$$

The sign of e, f, g is positive when it is in the same direction of the respective unit vector. From this equation, the position of point A is located.

There has been continuous discussions and great hesitation as to the choice of the particular point to be studied. The ideal point in this case should be such a point that the rotation and translation of the point should be invariant as to size and shape of the particular bone. That is, it should form some kind of a parameter so that the movement of the point in different bones can be compared. No one yet knows what such a parameter should be. The points considered include the geometric center, the mass center, points on the axis of rotation or points at some fixed distance from it, and the center of rotation. At present, particular attention is being

the target is oriented into a position when points C and D coincide. The second picture is taken when the points C and B coincide. Therefore in both pictures, the points C, D and B will form a straight line. Referring to Figure 24, let the unit vector

$$\hat{\vec{n}} = \hat{\vec{CB}} \times \hat{\vec{CD}}$$

$$\hat{\vec{m}} = \hat{\vec{CB}}$$

$$\hat{\vec{O}} = \hat{\vec{CD}}$$

Furthermore, let the perpendicular distance from point A to plane BCD be e, the perpendicular distance to line CD parallel to plane BCD be f, and the perpendicular distance to line BD parallel to plane BCD be g. Then because of the fact that the target is made in such a way that $\vec{CD}$ and $\vec{CB}$ are orthogonal to each other, it follows:

$$\vec{CA} = \hat{\vec{n}}(e) + \hat{\vec{m}}(f) + \hat{\vec{O}}(g)$$

The sign of e, f, g is positive when it is in the same direction of the respective unit vector. From this equation, the position of point A is located.

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directed toward the geometric center of the bone. This is being done by assuming that the center of area of the bone in the two x-ray pictures is the geometric center.

Work done to date on this part of the problem indicates that if the particular point in the bone is located from the x-ray, the equation for locating the coordinates of the point can actually be applied to find the translation of this point. Since the distance e, f, g are invariant to motion of the bone, therefore, once these distances are determined, they can be applied to locate the particular point at each new position of the bone.

Chapter 5.
Error Analysis

The merit of a system such as this one is judged to a great extent by its result. That is, one should bear in mind whether the results calculated from the measured data are in any way compatible with the actual motion. Therefore it is pertinent that one should evaluate the accuracy of the system in order that the results will not be mis-interpreted.

In this case I shall first of all proceed to analyze the probable errors in the measured quantity, then to discuss the inconsistency in the location of the reference axes, the location of the geometric center, and finally to discuss the accuracy of the system as a unit.

A) Measured Quantity.

Since the translation and rotation of a point in the bone cannot be measured directly, the measured quantities are the coordinates of the three target points. The translation and rotation are then computed from the measured quantities which necessarily involves a propagation of error.

It is assumed in this case that systematic errors are negligible and that all errors are random in nature. The random errors are due to two sources. One is due to the scanning system. It is estimated by the staff of the Photogrammetry Laboratory of M.I.T. using a calibrated meter bar that the scanning system measures to an accuracy of approximately ± 0.03 m.m. The second source of errors

comes from the target point. It is known that the target point which is the cross-section of a copper wire of approximately 0.5 m.m. in diameter has a finite area, and the coordinates of the point are recorded whenever the pointed indicator touches the cross-section of the point. This readily shows that the coordinates of the point in the X, Y direction can be off as much as 0.25 m.m., and because of the fact that the pointed steel rod is elastic, there certainly is some error introduced in the Z-coordinates. From twelve measurements of a target point under normal experimental condition, it is found that the standard deviation in all three coordinates is approximately ± 0.12 m.m. This shows that if the diameter of the wire is smaller, the error will also decrease. However, there is a practical limit to this, and we feel that any wire smaller than this will not only slow the experiment considerably, but would make the target point structually so weak that constant repairing has to be done. The above analysis indicates that the error due to the scanning system is negligible in comparison with the error due to the target point. Therefore in the actual error analysis, the error due to the scanning system can be neglected.

In the calculation of probable error of the computed quantity, the following error propagation formulas are being used[7]:

$$(A \pm a) + (B \pm b) = (A + B) \pm \sqrt{a^2 + b^2}$$

$$(A \pm a) - (B \pm b) = (A - B) \pm \sqrt{a^2 + b^2}$$

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[7] W. N. Bond, "Probability and Random Errors", Edward Arnold and Company, 1935.

$$(A \pm a)(B \pm b) = AB\left[1 \pm \sqrt{(\tfrac{a}{A})^2 + (\tfrac{b}{B})^2}\right]$$

$$\frac{(A \pm a)(B \pm b)}{(C \pm c)} = \frac{AB}{C}\left[1 \pm \sqrt{(\tfrac{a}{A})^2 + (\tfrac{b}{B})^2 + (\tfrac{c}{C})^2}\right]$$

$$A\,(1 \pm \frac{a}{A})^{k} = A^{k}\,(1 \pm k\frac{a}{A})$$

From these formulas we see that the errors propagate approximately to the square root of the sum of the squares except in the case of taking powers. Therefore, the more computation involved, the larger will be the error. This is the reason why we decide to use the equation for the location of point on x-ray for the calculation of translation. Admittedly, this equation is not as general as the other, but it involves less calculation, and consequently, the error will be less.

The calculation of the probable error in the case of rotation is more difficult and the result is less general. This is due to the fact that sine and cosine terms are involved. As they are sinusoidal curves, a look at their curves will show that for a given error in the Y term, the magnitude of error in the X termwill depend on the location of the point on the curve. For example, in the equation Y = sin X, for a given error in Y, the magnitude of error in X will be greater if X is near $\pi/2$ than if X is near 0 or π. However, the magnitude of probable error should be in the same order of magnitude as that of translation.

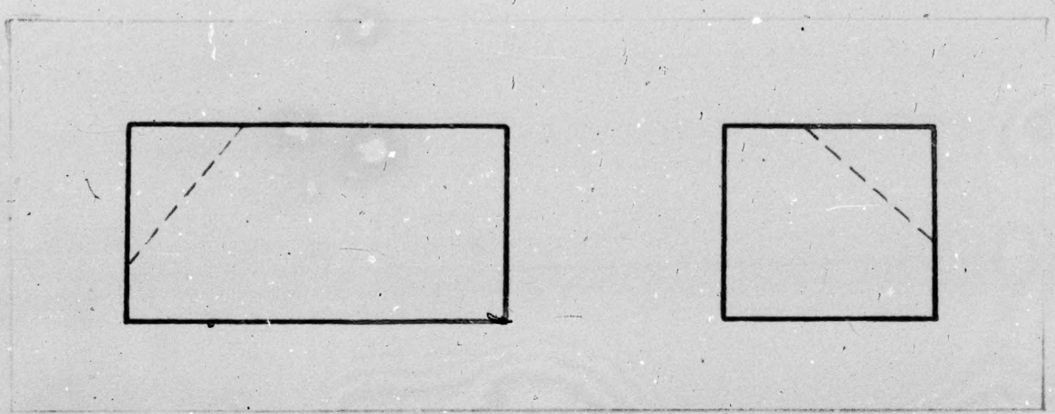
B) Reference Axes.

The justification for the use of the reference axes is based

on the assumption that the different motion of the bones referred to the reference axes can truly be compared. It is my belief that the use of our present reference axes will give logical and meaningful results. Whether this is true or not can only be judged by further experimental results. The error analysis in this case, therefore, is not on the evaluation of the validity of the reference axes, but on the method for the location of such axes. The method for the location of the reference axes is not satisfactory in the sense that the probable error involved is large and indeterminate. After careful evaluation, it is safe to say that the inconsistency arises from the assumption that the line joining the point G and H (referring to Fig. 9) is parallel to the reference x-axis. Because of the impossibility of locating the center of the lower end of the tibia without disarticulating the specimen, this indirect method has to be used. However, with an experienced operator working carefully, the error involved should not be greater than a few millimeters.

C) Geometric Center.

The method for the location of the geometric center as described previously is satisfactory for a symmetrical shape object as is shown in descriptive geometry, but for a non-symmetrical shape object, this is not the case. This is especially true in the case of the calcaneus. The x-ray pictures taken fail to show the irregularities which we know to exist in the calcaneus.



Front View Side View

Figure 25. Irregular Block Illustrating the Defect in the Location of the Geometric Center.

Consider a rectangular block with a corner cut off. Figure 25 shows that for x-ray pictures taken in these positions, the body will appear in the x-ray as a rectangular block. Thus, the geometric center located from these pictures will not be the true geometric center.

The investigation of the accuracy of the different components of the system indicates that the error involved in the determination of the reference axes and the location of the geometric center are largely indeterminate in nature. However, if the target is always fixed to the bone in the same position and the x-ray pictures are taken when the bone is always oriented in certain position, then the point located may not be the true geometric center, but will be a consistent point in the bone. We are then actually studying the translation and rotation of another point,

but the important thing is that they are the same point. It may be interesting if after the specimen is disarticulated, the center of the lower end of the tibia is located, and the reference axes so established are compared with that of the present method. Doing this for a few specimens would at least indicate the order of magnitude of the error.

Finally, one can say that this system is accurate enough to allow us to study the trend and pattern of motion of the bones in the foot.

Chapter 6.
Conclusion.

It is generally recognized in the medical field that the motion of the foot is complex and that there is a definite pattern of the type and range of motion involved. However, the work that has been done in this field is scarce and the method is usually not refined. Consequently, the results are merely an indication that such a trend and pattern do exist. The distinguishing feature of this approach lies in the rigorous treatment of the whole system as a technique for the study of the motion of the foot. All terms and all assumptions are defined explicitly so that there will not be any misunderstanding leading to the misinterpretation of results. We then have a basis for the evaluation of the result, and the result, if there is any, should be reliable.

Careful evaluation of the system also indicates that future efforts should be concentrated in the reduction of the amount of calculation, in the selection of reference axes, and the establishment of a parameter to facilitate comparison. The technique for measurement is accurate enough that further refinement would not improve the accuracy of the system to any great extent.

Although this system is limited to the study of the kinematical aspects of the motion, it is doubtful whether the study of the other kinematic and kinetic part of the motion can be done without prior knowledge of the type and range of motion. Therefore, it seems that this is the first step to the study of the motion of

the foot.

No conclusive result can be drawn at this time. The translation and rotation of a point in the bone can be used to fit into the equations of different types of curves to indicate the characteristics of the different types of motion. In this connection, there is indication that the center of rotation of the talus is outside of the talus, therefore in dorsi-plantar flexion, the talus rotates as well as translates.

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APPENDIX A

Sample of Data and Calculation

Appendix A

Data Sample

Experiment 10 March 7, 1958.

Point	X	Y	Z
E	246.11	73.25	229.82
F	244.83	73.25	220.58
G	208.27	105.27	220.44
H	199.40	412.87	229.15

The distance P is 6 m.m.

Note:

1. Ratio for Z-coordinates is 1:2. Gear No. 5.
2. All measurements are in millimeter.
3. Direction of Motion is from dorsal flexion (Position 1) to plantar flexion (Position 5).

TALUS

	Position	B X	B Y	B Z	C X	C Y	C Z	D X	D Y	D X
1.	E.R.*	132.84	363.08	237.90	120.97	397.20	245.75	162.48	409.70	248.23
1.	I.R.*	134.69	359.12	234.91	120.93	392.67	242.57	161.46	406.55	246.14
2.	E.R.	125.40	378.13	242.36	118.34	414.50	249.09	161.38	421.99	250.35
2.	I.R.	124.00	376.45	235.42	114.99	412.90	241.40	157.07	420.93	245.41
3.	E.R.	118.13	396.24	243.84	116.04	433.98	248.90	159.67	435.74	250.25
3.	I.R.	114.60	399.58	234.61	112.10	437.85	238.51	154.49	438.98	243.45
4.	E.R.	112.76	418.07	241.47	116.25	456.42	244.48	159.42	451.67	247.32
4.	I.R.	110.10	417.26	234.20	112.69	455.79	236.77	154.67	451.64	242.24
5.	E.R.	111.49	433.44	244.68	119.60	471.40	246.32	161.80	462.52	248.39
5.	I.R.	109.14	435.64	235.27	117.43	473.63	236.49	158.50	464.77	241.83

* E.R. - External Rotation

* I.R. - Internal Rotation

FIBULA

	B			C			D		
	X	Y	Z	X	Y	Z	X	Y	Z
1.	288.63	449.17	257.17	316.00	422.38	255.14	287.87	395.83	245.99
1.	286.61	449.48	258.18	313.99	422.90	256.51	286.84	396.08	245.99
2.	289.27	449.69	256.70	316.45	423.10	254.63	288.33	396.05	244.73
2.	286.06	449.35	258.44	313.35	422.76	257.04	286.56	395.87	246.23
3.	288.65	449.58	256.97	315.89	423.14	255.05	287.94	396.06	244.97
3.	285.28	449.40	258.73	312.76	422.79	257.54	286.07	395.95	246.48
4.	288.30	449.20	257.54	315.48	422.62	255.81	287.84	395.89	245.41
4.	284.77	449.67	258.82	312.22	423.02	257.55	285.70	396.33	246.56
5.	288.93	449.60	256.94	316.15	423.15	255.03	287.92	396.16	244.95
5.	285.70	449.77	258.25	313.09	423.25	256.81	286.28	396.30	245.98

CALCANEUS

		B			C			D		
		X	Y	Z	X	Y	Z	X	Y	Z
1.	E*	126.62	428.42	258.02	131.38	464.63	263.26	174.38	461.15	259.63
	I*	117.00	432.04	248.50	119.29	469.37	251.61	162.80	466.76	252.37
1.	E	127.03	420.92	257.23	129.96	456.92	263.30	173.08	455.67	259.78
	I	116.48	426.32	246.84	117.18	463.47	250.38	160.64	462.54	251.67
2.	E	128.68	446.14	260.16	137.96	482.32	263.85	180.10	473.12	259.56
	I	118.15	447.84	250.08	124.30	485.05	251.55	167.35	478.13	251.83
2.	E	124.21	442.88	255.42	132.22	478.92	260.01	174.94	470.50	257.46
	I	114.85	446.40	243.51	120.66	483.60	245.48	163.26	476.90	248.12
3.	E	128.86	464.22	258.05	141.98	499.43	260.40	182.85	484.71	257.18
	I	119.86	466.39	248.35	130.75	502.44	248.25	172.43	490.09	249.32
3.	E	125.03	466.82	251.47	138.79	501.77	254.44	179.35	485.89	253.74
	I	117.31	469.91	238.49	129.13	505.81	238.50	169.63	492.77	243.03
4.	E	133.96	483.41	254.31	152.61	516.35	255.64	190.62	495.05	253.94
	I	126.77	489.57	242.46	143.93	522.94	240.79	182.70	503.87	243.93
4.	E	128.50	483.87	248.20	146.24	516.95	249.98	184.62	496.15	250.65
	I	121.84	485.65	235.92	138.14	519.70	234.64	176.69	502.31	240.17
5.	E	141.05	496.00	255.64	162.98	526.82	255.84	198.56	501.64	253.64
	I	132.14	501.96	242.62	152.34	533.01	239.50	189.34	510.90	242.52
5.	E	136.29	498.80	247.84	158.27	529.53	248.57	193.85	504.18	249.30
	I	130.40	501.95	234.33	151.18	532.79	231.87	186.99	511.14	237.75

* Eversion

* Inversion

Transformation of Axes

Referring to Fig. 9 to Fig. 13

	X	Y	Z
E	246.11	73.25	459.64
F	244.83	73.25	441.16
G	208.27	105.27	440.88
H	199.40	412.87	458.30

$$\hat{\overrightarrow{FK}} = \hat{\overrightarrow{GH}} = \frac{-8.87\,\vec{i} + 307.60\,\vec{j} + 17.42\,\vec{k}}{\sqrt{(8.87)^2 + (307.60)^2 + (17.42)^2}}$$

$$= -0.02878\,\vec{i} + 0.99799\,\vec{j} + 0.05652\,\vec{k}$$

$$\overrightarrow{FE} = 1.28\,\vec{i} + 18.48\,\vec{k}$$

$$\overrightarrow{FE} \cdot \hat{\overrightarrow{FK}} = (-0.02878)\,(1.28) + (0.05652)\,(18.48)$$

$$= 1.00765$$

$$\hat{\overrightarrow{FK}}\,(1.00765) = (Lx - Fx)\,\vec{i} + (Ly - Fy)\,\vec{j} + (Lz - Fz)\,\vec{k}$$

$$Lx = 244.83 - 0.02878\,(1.00765) = 244.801$$

$$Ly = 73.25 + 0.99799\,(1.00765) = 74.25562$$

$$Lz = 441.16 + 0.05652\,(1.00765) = 441.21695$$

$$\hat{\overrightarrow{LE}} \times \hat{\overrightarrow{FK}} = \begin{vmatrix} i & j & k \\ 0.0708 & -0.0544 & 0.9960 \\ -0.02878 & +0.99799 & 0.05652 \end{vmatrix}$$

$$= -0.99707\,\vec{i} - 0.03266\,\vec{j} + 0.07223\,\vec{k}$$

$P = 6$ m.m.

$$0x' = 244.801 - (0.02878)6 = 244.62832$$

$$0y' = 74.25562 + (0.99799)6 = 80.24356$$

$$0z' = 441.21695 + (0.05652)6 = 441.55607$$

Translation of Axes.

$$X' = x - 244.63$$

$$Y' = y - 80.24$$

$$Z' = z - 441.56$$

Rotation of Axes

0' - x' axis

$$a_1 = -0.02878$$

$$a_2 = +0.99799$$

$$a_3 = +0.05652$$

0' - y' axis

$$b_1 = -0.99707$$

$$b_2 = -0.03266$$

$$b_3 = +0.07223$$

0' - z' axis

$$c_1 = 0.0708$$

$$c_2 = -0.0544$$

$$c_3 = 0.9960$$

$$x'' = -0.02878x' + 0.99799y' + 0.05652x'$$

$$y'' = -0.99707x' - 0.03266y' + 0.07223z'$$

$$z'' = 0.0708x' - 0.0544y' + 0.9960z'$$

Sample Calculation.

The coordinates of the target points B, C, D of the talus at position 1 - Internal Rotation are transferred to the new reference axes and the coordinates of point A are located.

Point B.

$x' = 134.69 - 244.63 = -109.97$

$y' = 359.12 - 80.24 = 279.88$

$z' = 469.82 - 441.56 = 28.32$

$x'' = -0.02878\,(-109.97) + 0.99799\,(279.88) + 0.05652\,(28.32) = 284.0829$

$y'' = -0.99707\,(-109.97) - 0.03266\,(297.88) + 0.07223\,(28.32) = 102.5525$

$z'' = 0.0708\,(-109.97) - 0.0544\,(297.88) + 0.9960\,(28.32) = 5.1953$

Point C.

$x' = 120.93 - 244.63 = -123.73$

$y' = 392.67 - 80.24 = 313.43$

$z' = 485.14 - 441.56 = 43.64$

$x'' = -0.02878\,(-123.72) + 0.99799\,(313.43) + 0.05652(43.64) = 318.8274$

$y'' = -0.99707\,(-123.73) - 0.03266\,(313.43) + 0.07223(28.32) = 116.2830$

$z'' = 0.0708\,(-109.97) - 0.0544\,(279.88) + 0.9960\,(28.32) = 5.1953$

Point D.

$x' = 161.46 - 244.63 = -83.20$

$y' = 406.55 - 80.24 = 327.31$

$z' = 492.28 - 441.56 = 50.78$

$x'' = -0.02878\,(-83.20) + 0.99799\,(327.31) + 0.05652\,(50.78) = 331.9167$

$y'' = -0.99707\,(-83.20) - 0.03266\,(327.31) + 0.07223\,(50.78) = 75.9341$

$z'' = 0.0708\,(-83.20) - 0.0544\,(327.31) + 0.9960\,(50.78) = 26.8806$